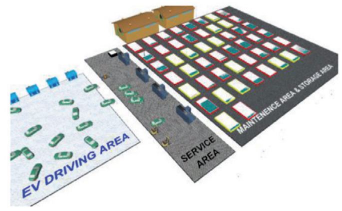
# Optimal operation of electric vehicle batteries in smart grids considering vehicle-to-grid technology




## Funcionamiento óptimo de baterías de vehículo eléctrico en redes inteligentes considerando la tecnología vehículo-a-red

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## RESUMEN

- En un futuro previsible se supone el despliegue a gran escala de vehículos eléctricos (EVs). La cuantiosa carga y descarga intermitentes reducen la capacidad y la vida de la batería debido a la tecnología vehículo-red (V2G) empleada en las redes inteligentes. Por eso, el modelado de la operación y mantenimiento de la batería es clave para evaluar la utilización de EVs. En este estudio, se analizaron el marco de aplicación V2G y el proceso general para definir el funcionamiento de la batería. Se estudió el modelo de operación y mantenimiento de la batería bajo tres estados de trabajo (esto es, carga, descarga y circulación) y un estado de fallo. La batería se deterioró a lo largo del tiempo y requirió un mantenimiento correctivo. Por algunas limitaciones dadas, se modeló y analizó el costo promedio a largo plazo aumentando y disminuyendo el proceso geométrico. La expresión analítica de la función de la tasa de costo se obtuvo del modelo propuesto de operación y mantenimiento. La correspondiente operación óptima se determinó también numérica o analíticamente para minimizar el costo promedio a largo plazo. En el modelo de operación y mantenimiento propuesto la batería fue reemplazada por una nueva idéntica tras el Nsimo fracaso. Se presenta un sistema de simulación numérica para ilustrar el modelo y método de funcionamiento optimizado. La amplia presentación de los resultados de la simulación muestran mejoras de rendimiento significativas usando el modelo propuesto, en comparación con el método convencional de carga/descarga coordinada por redes inteligentes. Puede ser aplicado en el sistema de gestión de la batería y resuelve los problemas de costos causados por el EVs.
- **Palabras clave:** Vehículo eléctrico, Batería, Vehículo-Red, Costo Medio, Proceso Geométrico.

## ABSTRACT

The large-scale deployment of electric vehicles (EVs) is anticipated in the foreseeable future. Heavy intermittent charging and discharging reduce battery capacity and lifetime because of the vehicle-to-grid (V2G) technology employed in smart grids. Thus, modeling battery operation and maintenance is key to assess the adoption of EVs. In this study, the V2G application framework and generalized process for determining battery operation were analyzed. An operation and maintenance model was studied under three battery working states (i.e., charging, discharging, and traveling) and one repairable failure state. The battery deteriorated over time and required corrective maintenance. For some given constraints, the long-run average cost was modeled and analyzed by increasing and decreasing geometric process. The analytical expression of the cost rate function was then derived based on the proposed operation and maintenance model. The corresponding optimal operation was also determined to minimize the long-run average cost. The battery in the proposed operation and maintenance model was replaced with a new and identical battery following the Nth failure. A numerical simulation system was presented to illustrate the optimized operation model and method. Extensive simlulation results, by using the proposed model and method, show significant performance improvements compared with conventional coordinated charging/discharging in smart grids. It can be applied in battery management system and solve the cost problems caused by EVs.

Keywords: Electric vehicle, Battery, Vehicle to grid, Average cost rate, Geometric process.

## 1. INTRODUCTION

The electrification of the transport sector through the adoption of electric vehicles (EVs) is considered to have the potential to relieve both global warming and impending world petroleum fuel energy shortages [1]. Given the numerous government-initiated programs that incentivize the purchase of EVs, more of them are expected to be connected to the power grid [2,3].

The increased use of EVs serves as distributed energy storage supply sources to stabilize the grid. Numerous studies have addressed vehicle-to-grid (V2G) technology by investigating the technical and commercial feasibility of providing ancillary services to the grid [4,5]. Fig.1 illustrates the detailed V2G design and implementation architecture. EVs are linked to smart grid by controllers, which implied that power flow, whether to or from EVs, is controlled by needs of the electric system via a real-time signal. For battery cars, the grid-connected batteries charge during low demand hours and discharge when power is needed. Each vehicle must have three required elements: a connection to the grid for energy flow, control connection for communication with control centers and the EV battery management system for optimal operation.

By applying V2G technology, the government aims not only to minimize the effect on the grid but also to balance the frequency fluctuation caused by the distributed power penetration. The power capacity of EVs normally ranges from 10 KW to 24 KW, which is negligible compared with the MW basis of the power

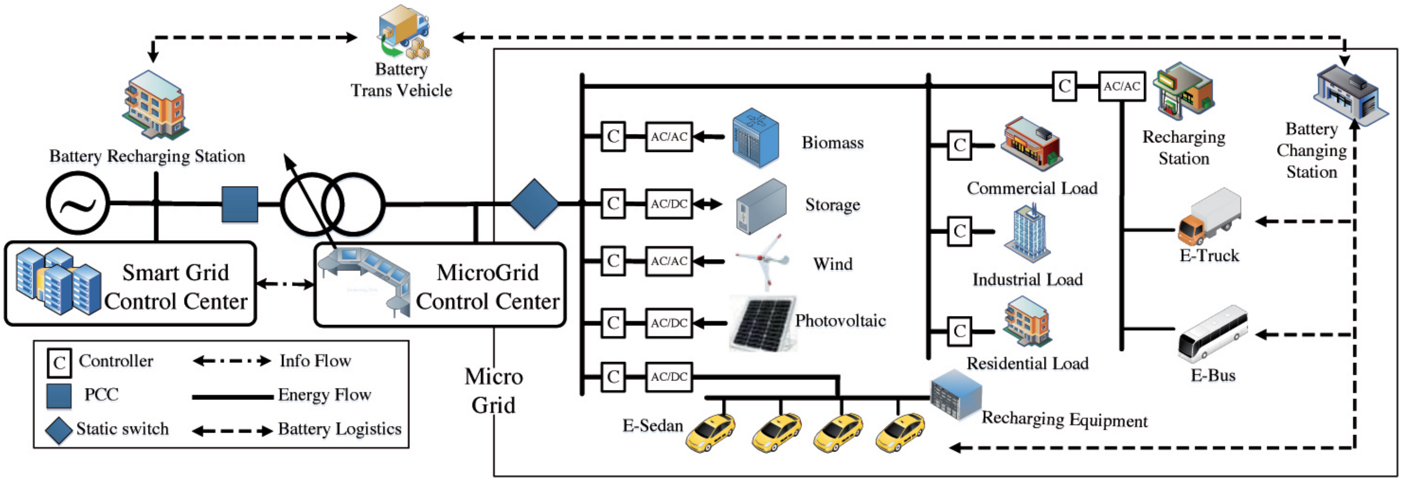



*Fig. 1: V2G implementation architecture*

fluctuation in smart grids. Thus, recharging and changing stations are constructed to fulfill large-scale power regulation services by aggregating a large number of EVs or batteries and by providing charging/discharging services. However, the EV battery has a life, and charging/discharging lead to cell imbalance, reduced battery capacity, and safety troubles. The cost of charging/discharging consists of battery aging or wear cost. The optimal operation of the EV battery should be considered in V2G application.

The capacity of EVs and the economic returns of participating in peak power, spinning reserve, and regulation markets have been explored in [6,7]. One of the weakest points of EVs is the battery. The life cycle of the EV battery is defined as the number of complete charge–discharge that it can sustain before its nominal capacity falls below 80% of its initial rated capacity [8]. For manufacturing asymmetries, charge–discharge lead to cell imbalance, reduced battery capacity, and safety troubles or a highly limited storage capacity of the full pack. Thus, the cost of charging/discharging consists of battery aging or wear cost. The issue of battery operation resulting from the life cycle cost has become important in power systems.

For the operation of an EV, the most advanced strategies rely on monitoring a measurable degradation process of the battery and on making maintenance-related decisions according to the monitored information. The battery management system is a critical component of EVs. As an electrochemical product, the battery acts differently under different operational and environmental conditions [9]. The narrow area within which lithium-ion batteries operate with safety and reliability necessitates their effective control and management. The efficient operating strategies based on system age and lifetime distribution are important [10]. However, building a degradation database is expensive, and most vehicles lack the capability of acquiring sensor-based information because of the non-existence of suitable quality characteristics. Many studies have focused on battery management and monitoring. In [11], a non-chemical-based and partially linearized input–output battery model was developed. The model was incorporated with the recovery effect for accurate lifetime estimation with a given set of typical loads at room temperature. The traditional battery maintenance method based on charge and discharge testing was proposed for sealed lead–acid batteries in [12]. This method involved applying a resistive charge to a set of batteries and adjusting a discharge current established by the manufacturer in battery rating tables. The proposed method was based on the discharge time versus discharge rate data given in the data sheets and measurements from the manufacturers. Cell-to-cell internal-impedance measurement was analyzed in [13]. These procedures all give the exact measures of the battery condition. However, the capacity test is not the most viable option for vehicles because the costs arising from vehicle nonavailability are high when the fleet of equipment to be maintained is large.

The battery operation strategy means improving the availability or economizing the operating costs of the battery such that the average cost rate is minimized. At the beginning, a new battery is installed. The battery is repaired as soon as it fails, after which the battery is presumably not as good as new. It is eventually replaced some time with a new and identical one. The replacement time is negligible. In traditional battery management system, always assumes that  the battery may only be in one of two possible states, either working or failed. And the optimal operation for the battery cycle cost caused by frequent charging/discharging has not yet been studied in detail in the existing literature. Furthermore, batteries for high-performances EVs should depend on high reliability. The reliability includes a long lifetime, high degree of safety, and energy regeneration capabilities. During the lifetime of a battery, the optimal operation and maintenance for the battery may have a great effect on the reliability. In this study, the effect of main-

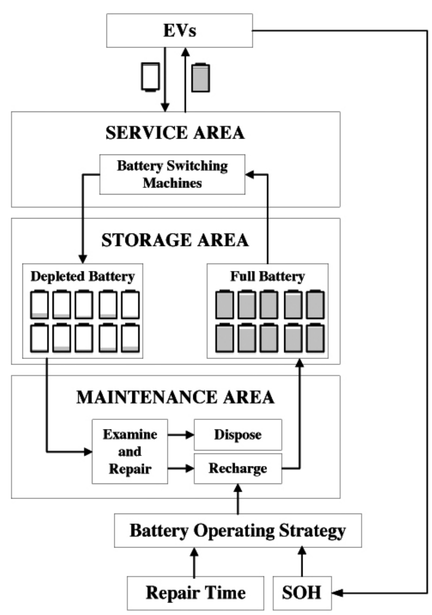



*Fig. 2: Generalized process for the battery operation*

tenance on the life cycle of the battery is studied in detail. The present study has the following objectives:

(1) To study the cost associated with the life cycle of the battery alongside the V2G system

(2) To establish a correlation between the battery life cycle cost and operation

(3) To determine the optimal operation of the battery given the conditions of frequent charging/discharging

The remainder of this paper is organized as follows. Sections 2 describes V2G design and implementation architecture. The proposed model and long-run average cost are presented in Section 3. Simulation results are given in Section 4. Section 5 concludes this paper.

## 2. APPLICATION FRAMEWORK

The proposed generalized process flow for battery operation is illustrated in Fig. 2. The process must operate over two different scales. The state of health (SOH) can be determined by evaluating the amount of capacity fade that has occurred in the battery. SOH estimations are conducted over the charge–discharge depending on the application and characteristics of a battery. Repair time estimations are conducted after the failure. The battery operation can be obtained according to the SOH and repair time. The optimal operation is applied to the maintenance, storage, and service areas.

## 3. MATHEMATICAL MODELS

### 3.1. BATTERY OPERATION AND MAINTENANCE MODEL

On the average, vehicles are parked 95% of the time. Thus, their batteries can be used to let electricity flow from the car to the power lines and back. There are three battery working states and one failure state, that is, the 1st-type working state (charging state), 2nd-type working state (discharging state), 3rd-type working state (transporting state), and failure state, respectively. A possible course of the battery is illustrated in Fig. 3.

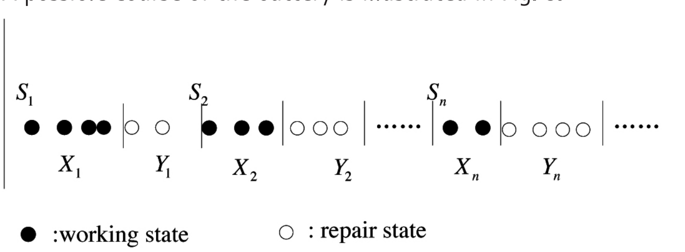



*Fig. 3: Possible course of the battery*

The time interval between the installation of a battery and the first replacement or two successive replacements is called a cycle. The time interval between the completion of the $(n\text{-}1)$th and $n$th failure repair is called the $n$th period of the cycle, and $n=1,2,\cdots,N\text{-}1$. The battery is replaced with a new identical one following the $N$th failure. During the lifetime of a battery, its performance or health tends to deteriorate gradually because of the irreversible physical and chemical changes that occur with usage and age until the battery is no longer usable or is dead.

Some definitions are given as follows:

1) Given two random variables, $X$ and $Y$, if

$$P(X>\alpha)\geq P(Y>\alpha)\ \text{for all real}\ \alpha \tag{1}$$

then $X$ is stochastically larger than $Y$, and $X\geq_{st} Y$ or $Y\geq_{st} X$. We also consider that a stochastic process $\{X_n,\ n=1,2,\cdots\}$ is stochastically decreasing if $X\geq_{st} X_{n+1}$, and a stochastic process $\{X_n,\ n=1,2,\cdots\}$ is stochastically increasing if $X\leq_{st} X_{n+1}$ for all $n=1,2,\cdots$.

2) $\{\zeta_n,\ n=1,2,\cdots\}$ is assumed to be a sequence of independent nonnegative random variables. If the distribution function of $\zeta_n$ is $P(\zeta_n\leq t) = U(a^{n-1}t),\ n=1,2,\cdots$, and $a$ is a positive constant, then $\{\zeta_n,\ n=1,2,\cdots\}$ is called a geometric process and $a$ is the ratio of the geometric process.

If $a>1$, then $\{\zeta_n,\ n=1,2,\cdots\}$ is stochastically decreasing:
$x_n >_{st} x_{n+1}, n=1,2,\cdots$

If $0<a<1$, then $\{\zeta_n,\ n=1,2,\cdots\}$ is stochastically increasing:
$x_n <_{st} x_{n+1}, n=1,2,\cdots$

3) A stochastic process $\{Z_n,\ n=1,2,\cdots\}$is composed of statistically independent nonnegative random variables. If the relationship $P(Z_n\leq z) = P(Z_1\leq a^{n-1}z)$ holds for $a>0$, then $\{Z_n,\ n=1,2,\cdots\}$ is called a geometric process.

Many factors contribute to the life cycle of a battery in a given application. SOH is an indication of the point that has been reached in its life cycle and a measure of its condition relative to that of a fresh battery. In this study, SOH is applied as the parameter of the life cycle of the battery. Let $X_1$ be the SOH of the battery after installation. In general, let $X_n$ be the SOH after the $(n\text{-}1)$th repair. Consequently, the successive charging or discharging capacity after repair decreases because of the deterioration. Let $Y_n$ be the repair time after the $n$th failure. In practice, a battery deteriorates because of the aging effect, accumulated wearing, and influence of the environment. On the contrary, the assumption that the repair time is not negligible is reasonable. The consecutive repair time may increase and tend to infinity. Then, $Y_n$ stochastically increases and tends to infinity. Therefore, a monotone process model should be the natural model for the battery.

The following probabilities and conditional probabilities are introduced to determine the distribution functions of $X_n$ and $Y_n$. The system state is defined as $S\,(t)$ at time $t$. The sets of working and failure states are $W=\{1,2,3\}$ and $F=\{4\}$, respectively. Initially, the new battery is in working state 1. When the battery fails, it is repaired. Let $t_n$ be the completion time of the $n$th repair, and let $s_n$ be the time of the $n$th failure. The transition probability from working state $i$ to the failure state is assumed to be given by

$$P\left(S\left(s_{n+1}\right)=4\middle|S\left(t_n\right)=i\right)=q_i \tag{2}$$

with $\sum_{i=1}^{3}q_i=1$. The transition probability from the failure state to the working state is also given by

$$P\left(S\left(t_n\right)=j\middle|S\left(s_n\right)=4\right)=p_j \tag{3}$$

with $\sum_{j=1}^{3}p_j=1$.

Let $X_1$ be the battery SOH after installation. In general, let $X_n$, $n=1,2,3,\cdots$, a life distribution $U\,(t)$ and $a_i\geq 1$ are assumed to exist; $i=1,2,3$, such that

$$P\left(X_1\leq t\right)=U\left(t\right) \tag{4}$$

and

$$P\left(X_2\leq t\middle|S\left(t_1\right)=i\right)=U\left(a_i t\right),i=1,2,3 \tag{5}$$

In general,

$$P\left(X_n \le t \middle| S(t_1)=i_1,\cdots,S(t_{n-1})=i_{n-1}\right) = U\left(a_{i_1}\cdots a_{i_{n-1}}t\right), i_j = 1,\cdots,3, j=1,2,\cdots,n-1 \tag{6}$$

Similarly, a life distribution $V(t)$ and $b \le 1$ are assumed to exist, given that only one failure state exists, such that $P(Y_n \le t|S(t_1) = i_1,\cdots,S|(t_{n-1}) = i_{n-1}) = V(b_{i_1}\cdots b_{i_{n-1}}t)$.

In general,

$$P\left(Y_n \le t \middle| S(t_1)=i_1,\cdots,S(t_{n-1})=i_{n-1}\right) = V\left(b_{i_1}\cdots b_{i_{n-1}}t\right) \tag{7}$$

The distributions of $X_n$ and $Y_n$ are determined.

$$\begin{aligned} P(X_2 \le t) &= \sum_{i=1}^{3} P\left(X_2 \le t \middle| S(t_1)=i\right)P\left(S(t_1)=i\right) \\ &= \sum_{i=1}^{3} p_i U(a_i t) \end{aligned} \tag{8}$$

In general,

$$\begin{aligned} &P(X_n \le t) \\ &= \sum_{i_1=1}^{3}\cdots\sum_{i_{n-1}=1}^{3} P\left(X_n \le t \middle| S(t_1)=i_1,\cdots,S(t_{n-1})=i_{n-1}\right) \times P\left(S(t_1)=i_1,\cdots,S(t_{n-1})=i_{n-1}\right) \\ &= \sum_{\sum_{i=1}^{3} j_i = n-1} \frac{(n-1)!}{j_1!\cdots j_3!} p_1^{j_1} p_2^{j_2} p_3^{j_3} U\left(a_1^{j_1} a_2^{j_2} a_3^{j_3} t\right) \end{aligned} \tag{9}$$

Similarly, it also can be obtained

$$\begin{aligned} &P(Y_n \le t) \\ &= \sum_{\sum_{i=1}^{3} j_i = n-1} \frac{(n-1)!}{j_1!\cdots j_3!} q_1^{j_1} q_2^{j_2} q_3^{j_3} V\left(b_1^{j_1} b_2^{j_2} b_3^{j_3} t\right) \end{aligned} \tag{10}$$

**Theorem** 1:

$$EX_n = \frac{1}{\lambda}\left(\frac{p_1}{a_1}+\frac{p_2}{a_2}+\frac{p_3}{a_3}\right)^{n-1}, \quad n=1,2,\cdots. \tag{11}$$

$$EY_n = \frac{1}{\mu}\left(\frac{q_1}{b_1}+\frac{q_2}{b_2}+\frac{q_3}{b_3}\right)^{n}, \quad n=1,2,\cdots. \tag{12}$$

**Proof.**

When $n=1$,

$$\begin{aligned} P(X_1 \le t) &= \sum_{i=1}^{3} U_i(t) \\ P(Y_1 \le t) &= \sum_{i=1}^{3} V_i(t) \end{aligned} \tag{13}$$

Therefore,

$$\begin{aligned} \frac{1}{\lambda} &= EX_1 = \sum_{i=1}^{3}\int_0^{\infty} t dU_i(t) \\ \frac{1}{\mu} &= EX_1 = \sum_{i=1}^{3}\int_0^{\infty} t dV_i(t) \end{aligned} \tag{14}$$

From (9), it can be obtained that

$$\begin{aligned} E(X_n) &= \int_0^{\infty} td\sum_{m=0}^{n-1}\sum_{i=1}^{3} U_i\left(a_1^{m_1}a_2^{m_2}a_3^{(n-1)-m_1-m_2}t\right)\frac{n!}{m_1!m_2!(n-m_1-m_2)!}p_1^{m_1}p_2^{m_2}p_3^{(n-1)-m_1-m_2} \\ &= \sum_{m=0}^{n-1}\sum_{i=1}^{3}\frac{n!}{m_1!m_2!(n-m_1-m_2)!}p_1^{m_1}p_2^{m_2}p_3^{(n-1)-m_1-m_2}\int_0^{\infty} tdU_i\left(a_1^{m_1}a_2^{m_2}a_3^{(n-1)-m_1-m_2}t\right) \\ &= \frac{1}{\lambda}\sum_{m=0}^{n-1}\sum_{i=1}^{3}\frac{n!}{m_1!m_2!(n-m_1-m_2)!}\left(\frac{p_1}{a_1}\right)^{m_1}\left(\frac{p_2}{a_2}\right)^{m_2}\left(\frac{p_3}{a_3}\right)^{(n-1)-m_1-m_2} \\ &= \frac{1}{\lambda}\left(\frac{p_1}{a_1}+\frac{p_2}{a_2}+\frac{p_3}{a_3}\right)^{n-1} \end{aligned} \tag{15}$$

Similarly, it follows from (10) that

$$E(Y_n) = \frac{1}{\mu}\left(\frac{q_1}{b_1}+\frac{q_2}{b_2}+\frac{q_3}{b_3}\right)^{n} \tag{16}$$

**Theorem** 2: For a real $t$, the following theorem shows that $\{X_1, X_2, \cdots X_n\}$ are a stochastically decreasing process and that $\{Y_1, Y_2, \cdots Y_n\}$ are a stochastically increasing process. For $n = 1,2,\cdots$ and $t > 0$, then

$$\begin{aligned} &P(X_{n+1} \le t) \\ &= \sum_{i_1=1}^{3}\cdots\sum_{i_n=1}^{3} p_{i_1}\cdots p_{i_n} U\left(a_{i_1}\cdots a_{i_n} t\right) \\ &\ge \sum_{i_1=1}^{3}\cdots\sum_{i_{n-1}=1}^{3} p_{i_1}\cdots p_{i_{n-1}}\left[\sum_{i_n=1}^{k} p_{i_n} U\left(a_{i_1}\cdots a_{i_{n-1}} t\right)\right] \\ &= \sum_{i_1=1}^{3}\cdots\sum_{i_{n-1}=1}^{3} p_{i_1}\cdots p_{i_{n-1}} U\left(a_{i_1}\cdots a_{i_{n-1}} t\right) \\ &= P(X_n \le t) \end{aligned} \tag{17}$$

Similarly,

$$C(N) = \frac{\text{Expected cost incurred in a renewal cycle}}{\text{Expected total energy}} = \frac{EC_c}{EP_p} \tag{18}$$

Given that $\{X_1, X_2, \cdots X_n\}$are stochastically decreasing and $\{Y_1, Y_2, \cdots Y_n\}$ are stochastically increasing, Theorem 2 shows that the battery operation and maintenance model is a monotone process.

### 3.2. LONG-RUN AVERAGE COST

In this section, the expression of the expected cost function is presented. Our objective is to determine the optimal $N^*$, such that the long-run average expected cost is minimized. $N^*$ also indicates that the battery is replaced with a brand new and identical one when the number of repairs reaches $N$. The explicit expression of the long-run expected cost per unit power energy is derived. The optimal $N^*$ is numerically determined by taking the following assumptions:

1) EV owners can neither produce any repair cost nor obtain any revenue during the delayed repair and the waiting period for the repair.
2) $C_r$, $C$, and $C_w$ are the failure repair cost rate, replacement cost, and working reward rate, respectively.

An optimal operation for the EV battery is applied, which means that the battery is replaced with a new identical one following the $N$th failure. The long-run average cost is then derived. Let $C(N)$ be the average cost rate of the battery under $N$. Thus, according to renewal reward theorem, $C(N)$ can be expressed as

$$P(Y_{n+1} \le t) \le P(Y_n \le t) \tag{19}$$

The total cost of a cycle is given by

$$C_c = C + C_r\sum_{n=1}^{N-1} Y_n - C_w \sum_{n=1}^{N} X_n C_{total} \tag{20}$$

$C_{total}$ is the rated capacitance. Thus, the expected cost rate $C(N)$ can be written as

$$\begin{aligned} C(N) &= \frac{C_r E\sum_{n=1}^{N-1}(Y_n) + C - C_w\sum_{n=1}^{N} E(X_n C_{total})}{\sum_{n=1}^{N} E(X_n C_{total})} \\ &= \frac{\frac{C_r}{\mu}\sum_{n=1}^{N-1}\frac{1}{b^{n-1}} + C}{C_{total}\frac{1}{\lambda}\sum_{n=1}^{N}\frac{1}{a^{n-1}}} - C_w \\ &= A(N) - C_w \end{aligned} \tag{21}$$

where

$$a = \left(\frac{p_1}{a_1} + \frac{p_2}{a_2} + \frac{p_3}{a_3}\right)^{-1}, \quad b = \left(\frac{q_1}{b_1} + \frac{q_2}{b_2} + \frac{q_3}{b_3}\right)^{-1} \tag{22}$$

$$A(N) = \frac{\frac{C_r}{\mu}\sum_{n=1}^{N-1}\frac{1}{b^{n-1}} + C}{C_{total}\frac{1}{\lambda}\sum_{n=1}^{N}\frac{1}{a^{n-1}}} \tag{23}$$

The optimal $N^*$ is determined clearly through analytical or numerical methods, such that $C(N^*)$or $A(N^*)$is minimized by evaluating first the difference of $A(N+1)$ and $A(N)$ .

$$\begin{aligned} &A(N+1) - A(N) \\ &= \frac{\left(\frac{C_r}{b^{N-1}\mu}\sum_{n=1}^{N} b^{n-1} + C\right)\frac{1}{a^{N-1}\lambda}\sum_{n=1}^{N} a^{n-1} - \left(\frac{C_r}{b^{N-2}\mu}\sum_{n=1}^{N-1} b^{n-1} + C\right)\frac{1}{a^{N}\lambda}\sum_{n=1}^{N+1} a^{n-1}}{C_{total}\frac{1}{a^{N}\lambda}\sum_{n=1}^{N+1} a^{n-1}\frac{1}{a^{N-1}\lambda}\sum_{n=1}^{N} a^{n-1}} \\ &= \frac{\frac{C_r}{\mu\lambda}\sum_{n=1}^{N} b^{n-1}\sum_{n=1}^{N} a^{n} + \frac{C}{\lambda}b^{N-1}\sum_{n=1}^{N} a^{n} - \frac{C_r}{\mu\lambda}\sum_{n=1}^{N-1} b^{n}\sum_{n=1}^{N+1} a^{n-1} - \frac{C}{\lambda}b^{N-1}\sum_{n=1}^{N+1} a^{n-1}}{C_{total}a^N b^{N-1}\frac{1}{a^{N}\lambda}\sum_{n=1}^{N+1} a^{n-1}\frac{1}{a^{N-1}\lambda}\sum_{n=1}^{N} a^{n-1}} \\ &= \frac{\frac{C_r}{\mu\lambda}H(N) - \frac{C}{\lambda}b^{N-1}}{C_{total}a^N b^{N-1}\frac{1}{a^{N}\lambda}\sum_{n=1}^{N+1} a^{n-1}\frac{1}{a^{N-1}\lambda}\sum_{n=1}^{N} a^{n-1}} \end{aligned} \tag{24}$$

where $H(N) = \sum_{n=1}^{N} a^n - \sum_{n=1}^{N-1} b^n$ . Given that the denominator of $A(N+1)-A(N)$ is always positive, the sign of $A(N+1)-A(N)$ is clearly the same as its numerator. The following auxiliary function is defined as

$$g(N) = \frac{\frac{C_r}{\mu\lambda}H(N)}{\frac{C}{\lambda}b^{N-1}} = \frac{C_r H(N)}{\mu C b^{N-1}} \tag{25}$$

The following lemma is straightforward.

**Lemma 1**:

$$A(N+1) \underset{<}{\overset{>}{=}} A(N) \Leftrightarrow g(N) \underset{<}{\overset{>}{=}} 1 \tag{26}$$

Lemma 1 shows that the monotonicity of $A(N)$ is determined by the value of $g(N)$. All results, including Lemma 1, hold for the battery operation and maintenance model.

The following results are also useful:

$$\begin{aligned} &H(N+1) - bH(N) \\ &= \sum_{n=1}^{N+1} a^n - \sum_{n=1}^{N} b^n - b\left(\sum_{n=1}^{N} a^n - \sum_{n=1}^{N-1} b^n\right) \\ &= (1-b)\left(\sum_{n=1}^{N} a^n - \sum_{n=1}^{N-1} b^n\right) + \left(a^{N+1} - b^N\right) \ge 0 \end{aligned} \tag{27}$$

According to (27), the following is evident:

$$\begin{aligned} &B(N+1) - B(N) \\ &= \frac{C_r H(N+1)}{\mu C b^N} - \frac{C_r H(N)}{\mu C b^{N-1}} \\ &= \frac{C_r b^{N-1}\left[H(N+1) - bH(N)\right]}{\mu C b^N b^{N-1}} \ge 0 \end{aligned} \tag{28}$$

Eq. (28) implies the following:

**Lemma 2**: $g(N)$ is non-decreasing with $N$.

Consequently, Lemma 1, together with Lemma 2, gives the analytic expression of $N^*$.

**Theorem 3**: An optimal $N^*$ for the deteriorating battery is determined by

$$N^* = \min\left\{N \middle| g(N) \ge 1\right\} \tag{29}$$

$N^*$ is unique if and only if $g(N^*)>1$.

The closed-form expression of the cost rate function makes the ensuing minimizing procedure easy to achieve.

## 4. SIMULATION RESULTS

The numerical example is discussed in this section. Lithium batteries are characterized by high specific energy, high efficiency and long life and have been used in electric vehicles. First, we test if the operating procedure of the battery in the EV agrees with a geometric process according to the theorems in [14]. The data set is originally studied with 220 Ah lithium-ion battery with 1C charging and discharging rates in Fig.4. In this study, the AnyLogic model template and MATLAB software are adopted. The agent-based modeling method in AnyLogic is imployed to construct our simulation system as Fig.2 . MATLAB software is then run to conduct the calculations.

The active agent in the model is EV, which may execute various behaviors. The EV agents in the simulation system are also modeled dynamically on traffic environment, which can help vi-

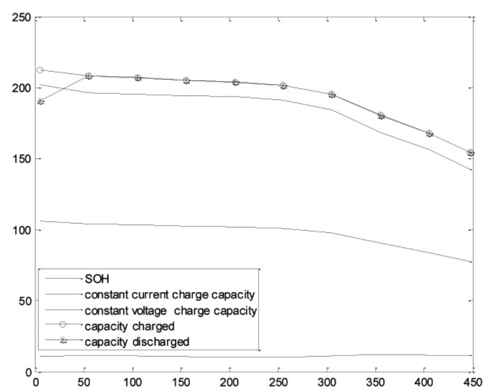



*Fig. 4: The data set with 220 Ah lithium-ion battery*

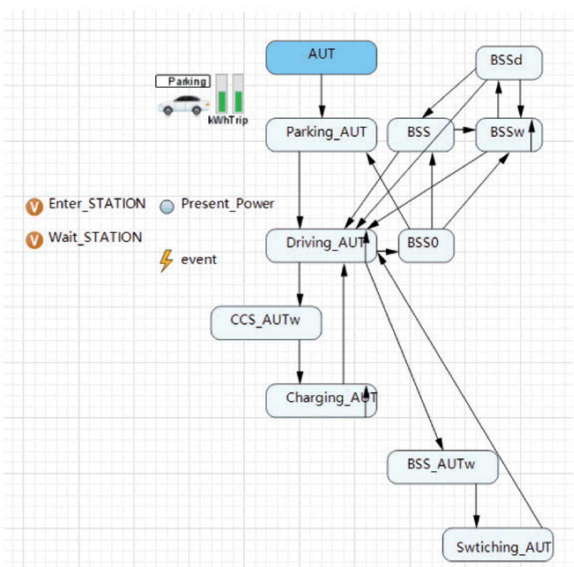
sualize the EVs that cause emerging phenomena, such as traffic congestions in charging, discharging, and traveling. The states for EV battery can be described as Fig.2, including traveling, served (charging/discharging) and maintained. The message ports and communication rules are embedded to enable interactions and communications among different agents. The statechart of the EV is shown in Fig. 5(a), where the sequence of vehicles for paring and charging/discharging is shown. Fig. 5(b) shows the visualized implementation.

To test whether the operating procedure agrees with a geometric process, the values $P_T^U$, $P_D^U$, $P_T^V$, and $P_D^V$ are calculated [14]. If a *p*-value is small, less than 0.5 say, the null hypothesis is rejected. The *p*-values $P_T^U$, $P_D^U$, $P_T^V$, and $P_D^V$ are illustrated in Table I. Hence, the operating procedure of the battery can be modeled with a geometric process.

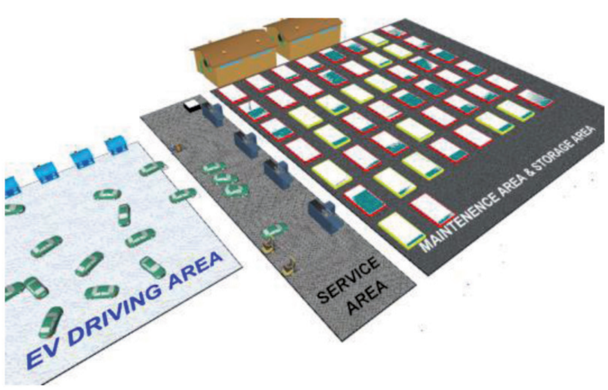
| $H_0$: the operating procedure of the battery is a geometric process. | | | |
|---|---|---|---|
| $P_T^U$ | $P_D^U$ | $P_T^V$ | $P_D^V$ |
| 0.9082 | 0.9207 | 0.9961 | 0.9207 |

*Table I. p-values of the testing*

In the following analysis, a numerical example is provided to explain how the optimal $N^*$ is determined. We assume that the




*(a) Statechart of the EV*




*(b) Visualized implementation*
*Fig. 5: Optimal operation simulation of the EV battery: (a) Statechart of the EV and (b) Visualized implementation*

lifetime distribution function $F_n(t)$ is known, and its parameters are given (estimated) with the values $a=1.0262$, $\lambda=102.485$, $b=0.9768$, $\mu=20$. The working reward rate $C_w$ can be obtained through weighted charging reward rated, discharging reward rated, and traveling reward rated.

The lifetime of a new battery has a Weibull distribution:

$$F_n(t)=1-e^{-(\lambda a^{n-1}t)^{\alpha}}, t>0 \tag{30}$$

where $\alpha$ and $\lambda$ are the parameters of the Weibull distribution [15].

For values of $N$, an optimal $N^*$ that minimizes the average cost function is searched. Fig. 5 shows the results, including the proposed method and the conventional coordinated charging/discharging, mentioned in [16]. The minimum of $C(N)$ is the expected cost, and the corresponding $N$ is the optimal operation for the EV battery. $N^* = 600$ is evidently the optimum at which $C(N)$ is minimized. $C(N^*)$=0.4101x$10^5$ is the minimum of the average cost (see Table II). Fig. 5 and Table II indicate that, when no optimal operation exists, the conventional cost $C_{con}(N)$ is approximately 1x$10^5$ higher than the proposed operating method. The conventional coordinated charging/discharging gives significant attention to the economic operation in smart grids without considering the battery-aging effect and accumulated wearing. This observation indicates that the proposed optimal operation can effectively reduce the cost. Thus, the battery can be scheduled as follows. The battery is replaced with a new one upon its 600th failure. Fig. 6 also shows the variety of the expected long-run average cost. The optimal $N^*$ can be determined uniquely.

| $N$ | 50 | 100 | 150 | 200 | 250 | 300 |
|---|---|---|---|---|---|---|
| $C(N)$ | 5.1958 x$10^5$ | 1.5910 x$10^5$ | 0.9946 x$10^5$ | 0.7526 x$10^5$ | 0.6238 x$10^5$ | 0.5456 x$10^5$ |
| $C_{con}(N)$ | 1.0059 x$10^5$ | 1.0099 x$10^5$ | 1.0219 x$10^5$ | 1.0259 x$10^5$ | 1.0259 x$10^5$ | 1.0459 x$10^5$ |
| $N$ | 350 | 400 | 450 | 500 | 550 | 600 |
| $C(N)$ | 0.4947 x$10^5$ | 0.4604 x$10^5$ | 0.4373 x$10^5$ | 0.4223 x$10^5$ | 0.4136 x$10^5$ | **0.4101** x$10^5$ |
| $C_{con}(N)$ | 1.0399 x$10^5$ | 1.0498 x$10^5$ | 1.0599 x$10^5$ | 1.0598 x$10^5$ | 1.0418 x$10^5$ | 1.0598 x$10^5$ |
| $N$ | 650 | 700 | 750 | 800 | 850 | 900 |
| $C(N)$ | 0.4112 x$10^5$ | 0.4166 x$10^5$ | 0.4262 x$10^5$ | 0.4401 x$10^5$ | 0.4585 x$10^5$ | 0.4818 x$10^5$ |
| $C_{con}(N)$ | 1.0658 x$10^5$ | 1.0878 x$10^5$ | 1.0758 x$10^5$ | 1.0857 x$10^5$ | 1.0877 x$10^5$ | 1.0957 x$10^5$ |
| $N$ | 950 | 1000 | 1050 | 1100 | 1150 | 1200 |
| $C(N)$ | 0.5105 x$10^5$ | 0.5452 x$10^5$ | 0.5865 x$10^5$ | 0.6355 x$10^5$ | 0.6931 x$10^5$ | 0.7606 x$10^5$ |
| $C_{con}(N)$ | 1.0796 x$10^5$ | 1.0896 x$10^5$ | 1.1157 x$10^5$ | 1.1257 x$10^5$ | 1.1156 x$10^5$ | 1.1296 x$10^5$ |
| $N$ | 1250 | 1300 | 1350 | 1400 | 1450 | 1500 |
| $C(N)$ | 0.8395 x$10^5$ | 0.9313 x$10^5$ | 1.0379 x$10^5$ | 1.1616 x$10^5$ | 1.3049 x$10^5$ | 1.4706 x$10^5$ |
| $C_{con}(N)$ | 1.1336 x$10^5$ | 1.1517 x$10^5$ | 1.1396 x$10^5$ | 1.1656 x$10^5$ | 1.1415 x$10^5$ | 1.1616 x$10^5$ |

*Table II. Comparative results for the proposed (C(N)) and conventional coordinated charging/discharging ($C_{con}(N)$)*

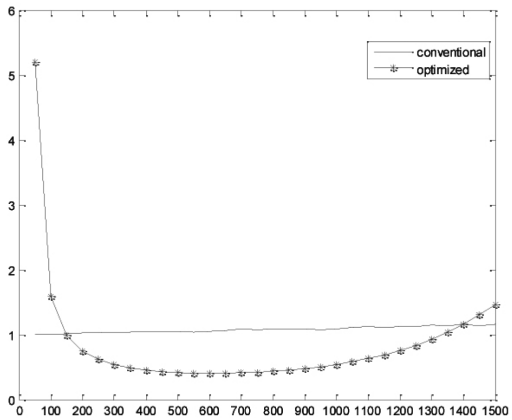



*Fig. 6: Average cost against N*

## 5. CONCLUSION

This study investigated the optimal operation for the EV battery in smart grids. A new battery operation and maintenance model with multiple states was studied. In this model, the battery-aging effect and accumulated wearing were considered. An optimal operation was then developed to minimize the long-run average cost. The simulation results confirmed that the optimal operation had an economical profile. The following are the main conclusions of this study:

(1) An operation and maintenance model was studied for the EV battery because of the V2G system. According to the results of the simulation, this model has the potential to provide economic benefits and safety benefits to the entire smart grid and to further improve energy management.

(2) The explicit expression of the long-run average cost was derived. The battery could be replaced with a new and identical one following the *N*th failure, which would significantly help reduce the battery cost. Thus, the model and the method can be applied to solve the challenging battery cost problem caused by the popularization of EVs.

(3) The proposed model and method were non-chemical, and building a degradation database was not necessary. Thus, they were more realistic and reasonable. The proposed model and method could be used for battery management and demand response because of their simplicity and flexibility.

The proposed model and method are suitable for the development of battery management used in EVs, which are implemented to achieve a further improvement in battery performance and life time.

Our work depends on accurate SOH estimation. As a future work we intend to investigate the bivariate model whenever SOH drops to a predetermined critical thereshold. Then the optimal operation method for EV batteries could not noly extend battery life time but also improve the EV reliablility.

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## APPRECIATION

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