Novel method of 3D slope stability analysis and its engineering application

Nuevo método de análisis de estabilidad de taludes en 3D y su aplicación en ingeniería

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RESUMEN

- Es difícil dar un resultado de evaluación definitivo cuando se utiliza el método de probabilidad difusa para evaluar la pendiente tridimensional (3D) y para desarrollar un método efectivo es necesario para evaluar la estabilidad de un gran número de ingeniería en taludes 3D. Para resolver este problema, se propone un nuevo método de análisis de estabilidad de taludes en 3D basado en el grupo de renormalización y la función de la superficie de respuesta. Después de determinar la probabilidad de inestabilidad crítica de la pendiente 3D basada en la teoría del grupo de renormalización, se emplea el método de superficie de respuesta para construir una función que resuelva la probabilidad de inestabilidad difusa de la pendiente 3D. De este modo, la estabilidad de la pendiente 3D puede determinarse comparando los valores de probabilidad de inestabilidad crítica y difusa. Cuando la probabilidad de inestabilidad difusa es menor que la probabilidad de inestabilidad crítica, la pendiente 3D se encuentra en un estado estable, de lo contrario se encuentra en un estado inestable. Este nuevo método para el análisis de estabilidad de taludes en 3D es verificado mediante la ingeniería de taludes de vertederos de minas. Los resultados muestran que el nuevo método propuesto es preciso y fiable. Este estudio resuelve el problema de la evaluación cuantitativa de la estabilidad de taludes en 3D, y las conclusiones obtenidas en el estudio son de gran valor de referencia para la práctica directa de ingeniería semejante.
- **Palabras clave:** Pendiente 3D, Grupo de renormalización, Probabilidad difusa, Análisis de estabilidadx.

ABSTRACT

It is difficult to give a definitive evaluation result when using the fuzzy probability method to evaluate the three-dimension (3D) slope and to develop an effective method is necessary to evaluate the stability of a large number of 3D slope engineerings. To solve this problem, a novel method of 3D slope stability analysis is proposed based on renormalization group and response surface function. After determining the critical instability probability of 3D slope based on the theory of renormalization group, the response surface method is employed to construct a function to solve the fuzzy instability probability of the 3D slope. Then the stability of the 3D slope can be determined by comparing the critical and fuzzy instability probability values. When the fuzzy instability probability is less than the critical instability probability, the 3D slope is in stable state, otherwise it is in unstable state. The novel method for 3D slope stability analysis is verified by the mine dump slope engineering. Results show that the proposed novel method is accurate and reliable. This study solves the problem of quantitative evaluation of 3D slope stability and the conclusions obtained in the study are of important reference value to direct similar engineering practice.

Keywords: 3D slope, Renormalization group, Fuzzy probability, Stability analysis.

1. INTRODUCTION

Due to the diversity and randomness properties of geotechnical media, the fuzzy probability method is often used to evaluate and analyze the stability of a three-dimension (3D) slope engineering. The fuzzy probability method is usually based on the limit equilibrium method to construct a functional function to characterize the stability of the slope. The 3D slope problem is usually simplified as a plain slope treatment, and the mechanical balance of the slope section is used to solve the problem [1-3]. However, the 3D slope engineering often there are obvious 3D deformation characteristics during the sliding failure process, and this is often accompanied by significant interactions among these sliding bodies in different spatial regions in the 3D slope [4-6].

Since the fuzzy probability method ignores the interactions among the different sliding bodies, it usually makes poor evaluation results of 3D slope stability. In addition, because there is no critical probability value to distinguish between stability and instability, it is difficult to give a definitive evaluation result when using the fuzzy probability method to evaluate 3D slope stability. With the rapid development of economic construction and infrastructural facilities in China, a large number of 3D slope projects came into being, so an effective method for 3D slope stability evaluation is urgently needed.

Therefore, to solve the above-mentioned problems, based on renormalization group and response surface function, a novel method is proposed to analyze 3D slope stability of the mine dumping in this study.

2. STATE OF THE ART

Recently, many scholars have discussed the fuzzy stability of the slope. For example, considering the influence of fuzzy random factors, Tan et al. put forward an improved fuzzy point estimation method and applied it to the reliability analysis of a slope [7]. Since the parameters affecting slope stability are both fuzzy and random, the concept and numerical characteristics of fuzzy random variables have been reported [8-9]. Giais et al. used convex set and first-order reliability methods to analyze the fuzzy random reliability of the slope [10]. Park et al. expressed the uncertain parameters of a slope as fuzzy numbers and fuzzy sets, and used Monte Carlo simulation technology to analyze the fuzzy reliability of the slope [11]. Tan et al. discussed the fuzzy distribution of slope reliability based on parameter fuzziness and randomness [12]. Based on classical reliability theory, Jia & He solved the slope reliability under different cut set levels [13]. According to the statistical results of a large number of slope engineerings, Jiang & Xu obtained the fuzzy random reliability evaluation index for the slope stability [14].

With the development of computer technology, some scholars combine numerical method with fuzzy probability to analyze the slope stability. For example, Jiang et al. solved the fuzzy instability probability of a slope by using fuzzy finite element method [15]. Wang et al. used fuzzy measure method to solve the fuzzy reliability index of each point at a slope [16]. Tan et al. transformed the fuzzy variable into random variable by the equivalent transformation method and studied the method of solving the fuzzy failure probability and the fuzzy random reliability index by using the probability integral method [17]. Most scholars usually simplified the mechanical equilibrium of a section of a slope instead of analyzing the overall stability of the slope by using the limit equilibrium method. Due to the complexity of the 3D slope sliding failure, it is difficult to perform a comprehensive evaluation of the overall stability of the 3D slope by the above-mentioned methods.

In view of the advantages of renormalization group theory in the study of critical problem, Gao et al. studied the self-organizing characteristics and critical conditions of rock failure process using renormalization group theory [18]. Zhou et al. established the renormalization group model of red-bed soft rock and calculated the percolation threshold to judge the stability of this soft rock [19]. Xue et al put forward an improved renormalization group model and revealed the critical behavior of rock deformation process [20]. Using renormalization group theory, Zhang et al. studied the relationship between deformation mechanism and permeability of brittle rock [21]. Gu et al. established a global damage model of prism element based on renormalization group method and studied the critical damage probability of concrete [22]. Guo & Liu studied the critical instability failure of coal pillar by using renormalization group theory [23]. Zhang et al. studied the critical stability of roof system of coal pillar by means of probability analysis and renormalization group theory [24]. All these studies demonstrate the applicability of renormalization group in solving critical problems.

The essence of slope progressive failure is the process that the basic components of the slope system undergo a series of instability and failure transfer, which ultimately leads to the overall macroscopic instability of the slope. The fixed point of the renormalization group is the critical failure probability of the slope. Therefore, the one-dimension, two-dimension and three-dimension renormalization group models of the slope were established in this study, and the unstable fixed points of the three slope models were solved respectively, which were regarded as the critical points for the overall instability of the slope. At the same time, parameter inversion analysis was used to reduce the fuzziness of rock and soil parameters, and the response surface method and numerical method were used to construct the slope stability function and the membership function, which represented the whole failure of the 3D slope. By comparing the instability probability with the critical instability probability, the 3D stability of the dump slope was evaluated synthetically. This is an effective quantitative evaluation of 3D slope stability to consider the 3D deformation effects and to give a definitive evaluation result than other methods.

The remainder of this study is organized as follows. In Section 3, the novel method for 3D slope stability analysis is proposed based on renormalization group and response surface function. In Section 4, the novel method is verified by a dump engineering practice and the research results are discussed. Section 5 summarizes the conclusions.

3. METHODOLOGY

3.1. RENORMALIZATION GROUP METHOD

A renormalization group is a mathematical tool that examines changes in physical systems at different length scales. The change in scale is called the "scale conversion". The renormalization group has a close relationship with "scale invariance" and "conformal invariance". The conformal invariance includes scale transformations, all of which is related to self-similarity. In renormalization theory, the system is self-similar to a smaller scale on a certain scale, and the variables of the system are described by the interaction between the system components.

The renormalization group was first applied to quantum field theory, which is an effective tool to solve critical problems. It is a method based on scaling invariance to observe the variation law of physical quantities by changing the scale in the system. Suppose the value of parameter a is changed from P to P' because the scale is magnified n times. If the scaling function is denoted as f_n according to reference [19], then the relation between P and P' is:

$$P' = f_n(P) \tag{1}$$

If the scale is magnified n times, then

$$P'' = f_n(p') = f_{n\times n}(P) \tag{2}$$

By changing formula (2) into a general relation, its transformation function f has the following properties:

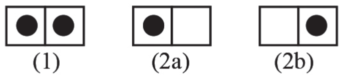
$$f_a \times f_a = f_{ab} \tag{3}$$

When using renormalization group theory to study the critical probability of slope stability, the following basic assumptions are made: (a) When the basic unit in the original package is completely destroyed, the original package is destroyed. (b) The probability of instability transfer from unstable element to its adjacent element is the same, and the energy loss in the transmission process can be ignored. (c) The instability transfer occurs only in the original package.

3.1.1. One-dimension renormalization group model

The slope system is divided into n segments and each segment is a basic unit. The whole stability of slope is related to the stability of basic unit. When n is large enough, the basic unit is small enough. In one-dimensional renormalization group model, two basic units are primary package, two primary packages constitute a secondary primary package, and two secondary primitive packets constitute a tertiary primary package. As shown in Fig. 1, the model diagram of the tertiary primary package is given.

(a) The tertiary primary package of one-dimensional renormalization group



(b) Failure of primary package

Fig. 1: One-dimension renormalization group model

It is assumed that the instability probability of the basic unit is P_0 and it satisfies the quadratic Weibull distribution:

$$P_0(F)=1-\exp\left[-\left(\frac{F}{F_0}\right)^2\right] \tag{4}$$

where F is the action value of instability index, and F_0 is the reference value of instability index.

According to the assumption, there are two cases of primary package instability. As shown in Fig. 1b, both of the basic units in the primary package are unstable in the first case. In the second case, there is a basic unit that is unstable in the primary package, and another basic unit is unstable due to the instability transfer. So the instability probability of the primary package is:

$$P_1=P_0^2+2P_0(1-P_0)P_{ab} \tag{5}$$

where P_1 is primary package instability probability, and P_{ab} is the conditional probability of unstable transfer of one base unit to its adjacent units. According to reference [25], P_{ab} can be expressed as

$$P_{ab}=\frac{P_b-P_a}{1-P_a} \tag{6}$$

where P_b is the instability probability of the adjacent basic unit after the failure of the basic unit, and P_a is the instability probability of the adjacent basic unit before the failure of the basic unit. After the failure of the basic unit, the instability index of adjacent basic unit is $2F$:

$$P_b=P(2P)=1-(1-P_0)^4 \tag{7}$$

In summary, the instability probability of the original package is as follows:

$$P_1=2P_0\left[1-(1-P_0)^4\right]-P_0^2 \tag{8}$$

According to the recursive relation of logistic one-dimensional mapping, the instability probability of n-order original packet can be obtained as follows:

$$P_n=2P_{n-1}\left[1-(1-P_{n-1})^4\right]-P_{n-1}^2 \tag{9}$$

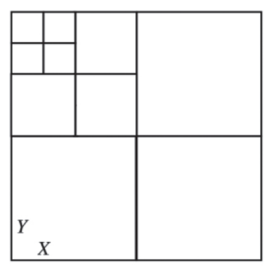
The formula (8) is one-dimensional renormalization group equation for slope instability probability, which is written as a function as follows:

$$f(x)=2x\left[1-(1-x)^4\right]-x^2 \tag{10}$$

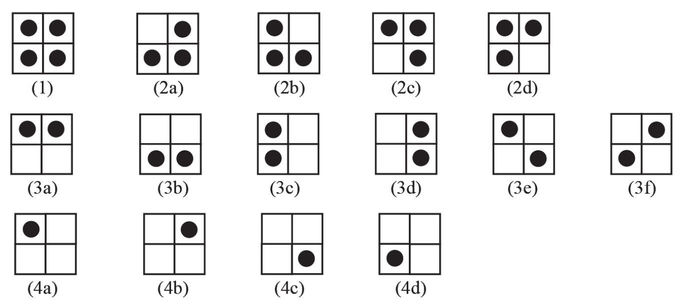
The upper formula has three fixed points in the range of [0,1], which are 0, 0.206, and 1, respectively. According to the criterion, when $\left|\frac{df(x)}{dx}\right|_{x}>1, x$, x is unstable point, otherwise it is stable point. The result shows: when $x=0$, there are no basic elements in the slope and the slope is stable as a whole. When $x=1$, each basic unit in the slope is unstable and the whole slope is in a state of ultimate instability. $x=0.206$ is the one-dimension critical instability probability of the slope. When $x<0.206$, the instability element exists in the slope, but the whole slope develops to the stable state. When $x>0.206$, most of the basic units in the slope are unstable, and the whole slope develops to the unstable state.

3.1.2. Two-dimension renormalization group model

As shown in Fig. 2, the slope system can be divided into $n\times n$ basic units, four basic units constitute one primary package, and four primary packages constitute one secondary primary package. Fig. 2 is the third-order primary package of two-dimensional renormalization group model.



(a) The tertiary primary package of two-dimensional renormalization group



(b) Failure of primary package

Fig. 2: Two-dimension renormalization group model

In accordance with the one-dimensional renormalization group model, assuming that P_0 is the instability probability of the basic unit, the instability probability of the primary package can be expressed as:

$$\begin{aligned}P_1=&P_0^4+C_4^3P_0^3\left[1-P_0-(1-P_0)^{16}\right]+C_4^2P_0^2\left[1-P_0-(1-P_0)^4\right]^2+C_4^2C_2^1P_0^2\left[1-P_0-(1-P_0)^4\right]\left[(1-P_0)^4-(1-P_0)^{16}\right]\\&+C_4^1C_3^1P_0\left[(1-P_0)^{16/9}-(1-P_0)^{16}\right]\left[1-P_0-(1-P_0)^{16/9}\right]^2+C_4^1C_3^1P_0\left[1-P_0-(1-P_0)^{16/9}\right]\left[(1-P_0)^{16/9}-(1-P_0)^4\right]^2\\&+C_4^1P_0\left[1-P_0-(1-P_0)^{16/9}\right]^3+C_4^1C_3^1C_2^1P_0\left[1-P_0-(1-P_0)^{16/9}\right]\left[(1-P_0)^{16/9}-(1-P_0)^4\right]\left[(1-P_0)^4-(1-P_0)^{16}\right]\end{aligned} \tag{11}$$

According to the recursive relation of logistic mapping, the instability probability of *n*-order original packet can be obtained as follows:

$$P_n = f\left(P_{n-1}\right) \tag{12}$$

So the critical instability probability of the slope obtained by using the two-dimensional renormalization group model is 0.1707.

3.1.3. Three dimension renormalization group model

In three-dimensional case, one primary package consists of eight basic units and one secondary primary package consists of eight primary packets. The model is shown in Fig. 3.

In accordance with the method of analyzing one-dimension and two-dimension problems, the instability probability equation of *n*-order original packet can be obtained according to the recursive relation of logistic mapping. The critical instability probability of slope is 0.1599, which is obtained by three-dimensional renormalization group model.

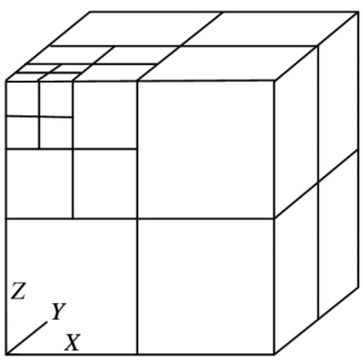


Fig. 3. Three-dimension renormalization group model

3.2. FUZZY PROBABILITY ANALYSIS METHOD

In slope stability analysis, the function of slope stability can be described as $Z(x_i)$, in which:

$$Z\left(x_i\right) = g\left(x_1, x_2, \cdots, x_{n-1}, x_n\right) = F_s - 1 \tag{13}$$

where, $x_1, x_2, \ldots, x_{n-1}, x_n$ are the uncertain factors affecting slope stability, and F_s is the slope safety factor.

According to fuzzy probability [17], the expression of fuzzy failure probability of slope is as follows:

$$P_f = \int_{-\infty}^{+\infty} \mu_A\left(z\right) f\left(z\right) dz \tag{14}$$

where P_f is the fuzzy instability probability of slope, $\mu_A(z)$ is the membership function of the slope stability function, and $f(z)$ is the probability density function of the slope stability function.

3.2.1. Statistical characteristics of uncertain variables

The statistical characteristics of uncertain variables affect the accuracy of evaluation results. The parameter inversion analysis method is adopted to determine the range and statistical characteristics of uncertain variables affecting slope stability by using $FLAC^{3D}$ technique.

First of all, the range of given uncertain variable is adjusted according to the actual survey results by conducting $FLAC^{3D}$ numerical simulation. Secondly, the range of the uncertain variable is given on the basis of the agreement between the slope deformation signs and the actual slope deformation signs in the simulation results. The range of uncertain variables obtained is $[x_{iL}, x_{iR}]$. In addition, assuming that the uncertain variables follow the normal distribution, and the confidence of the determined interval is 85%, the mean value μ_{x_i} and mean square deviation σ_{x_i} can be expressed as

$$\mu_{x_i} = \frac{x_{iR} + x_{iL}}{2} \tag{15}$$

$$\mu_{x_i} - 1.44\sigma_{x_i} = x_{iL} \tag{16}$$

3.2.2. Construction the function of slope stability

According to the limit equilibrium method, the function of slope stability is usually constructed by using the simplified mechanical equilibrium on a section of the slope. The premise of applying this method is that the slope can be simplified as a plane strain problem. Because of the obvious 3D deformation characteristics of the mine dump, the overall stability of the dump is closely related to the interactions among various elevation platforms, it cannot be reduced to a plane strain problem. Therefore, the fuzzy probabilistic method based on the limit equilibrium method cannot accurately evaluate this dump slope stability.

In this study, based on numerical simulation, the approximate original function obtained by response surface method is used to replace the slope stability function for fuzzy probability analysis. The specific steps are as follows:

(1) The quadratic polynomial with no crossover term is selected as the response surface function.

(2) Based on the range of the uncertain variable, the Bucher method is used to determine the test point. The sampling radius is $f\sigma_i$, f is deviation coefficient, and its value is 2, $\sigma_i = \frac{x_{iR} - x_{iL}}{2}$.

(3) The stability function values of each test point are calculated by $FLAC^{3D}$, and the undetermined coefficients in the response surface function are obtained by solving the linear equations. Thus the approximate function can be expressed as:

$$Z\left(x_i\right) = g\left(x_1, x_2, \cdots, x_{n-1}, x_n\right) = F_s - 1 = G\left(x_i\right) \tag{17}$$

3.2.3. Probability density function solution

The specific form of the membership function directly affects the evaluation results of slope stability. The form of the membership function should be similar to the fuzzy distribution characteristics of the object studied. An arbitrary stability function value Z_i, when n is large enough, its membership degree μ_{z_i} to the unstable event A can be approximated as the unstable membership frequency P_{z_i}.

$$\mu_{z_i} \approx P_{z_i} = \frac{Z_i \in A}{n} \tag{18}$$

where n is the number of samples whose stability function value is Z_i in a fuzzy statistical sample.

The results show: when $Z_i<0$, the variation rate of μ_{z_i} is larger than that when $Z_i>0$. Therefore, the membership function form used in this study is quadratic polynomial type.

For the slope engineering, the closer instability membership is to 1, the higher possibility of slope instability is. Conversely, the closer the instability membership is to 0, the smaller the probability of slope instability is. Combined with the related specifications, the membership function may be expressed in the general form:

$$\mu_A(z)=\begin{cases}1, & z\le -0.1\\ 8.33z^2-4.167z+0.5, & -0.1<z<0.2\\ 0, & z\ge 0.2\end{cases} \quad (19)$$

Assuming that the stability function is normally distributed, the mean value μ_z and the variance σ_z of the stability function can be obtained by the center point method as follows:

$$\mu_z = G(\mu_x) \quad (20)$$

$$\sigma_z=\left[\sum_{i=1}^{4}\left(\frac{\partial G}{\partial x_i}\Big|_{\mu_x}\sigma_{x_i}\right)^2\right]^{\frac{1}{2}} \quad (21)$$

where μ_{x_i} is the mean value of x_i, and σ_{x_i} is the mean square deviation of x_i.

Thus the probability density function can be expressed as:

$$f(z)=\frac{1}{\sqrt{2\pi}\sigma_z}\exp\left[-(z-\mu_z)^2/(2\sigma_z^2)\right] \quad (22)$$

Combining the three formulas (19), (22), and (14), the fuzzy instability probability P_f can be calculated. Comparing the critical instability probability P_L with the fuzzy instability probability, if $P_f > P_L$, the slope is unstable, otherwise the slope is stable.

4. ENGINEERING VERIFICATION ANALYSIS

4.1. ENGINEERING BACKGROUND

The Guangyuan dumping site is located in the middle section of the Yanshan Mountains in Chengde City, China. The area is a tectonic denuded landform. The terrain near the dumping site is high in the north and low in the south, and the mountain is relatively slow.

The horizontal area of this site is about 5600 m^2. The mine dump is placed on a moderately weathered gneiss with an inclination angle of about 15°. There is a weak bedding with a layer thickness of 2.0 m between the mine dump and the bedrock. The mine dump is composed of six different elevation platforms. The computational model is shown in Fig. 4. The physical and mechanical parameters of the model are shown in Table I.

The following are the deformation characteristics of Guangyuan mine dump:

(1) The maximum vertical displacement appears in No. 3 and No. 5 elevation platforms, which are formed by consolidation settlement.

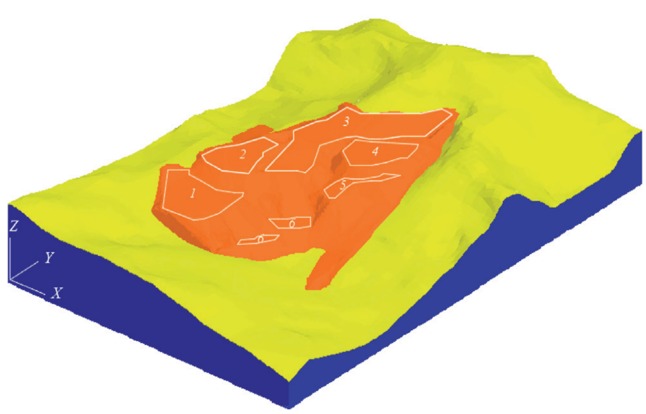


Fig. 4: The computational model of Guangyuan mine dump

Name	Bulk modulus (MPa)	Shear modulus (MPa)	Internal friction angle (°)	Cohesion (kPa)	Tensile strength (kPa)
Mine dump	55.6	18.5	27	50	2.0
Weak bedding	18.0	3.57	12	15	0.1
Bedrock	9800	3760	56	10000	2200

Table I Physical and mechanical properties of the computational model

(2) The horizontal displacements of No. 1, No. 3, and No. 5 elevation platforms are larger, and the whole platform develops towards the sloping direction. At the tops of elevation platforms No. 1, No. 3, and No. 5, the tensile damage was caused by the settlement of self-weight consolidation, and a crack of about 8.0 cm in width was produced there.

(3) Slippage occurs at steep ridges and steep slopes of No. 1, No. 3, No. 5, and No. 6 elevation platforms.

4.2. STATICAL CHARACTERISTICS OF UNCERTAIN VARIABLES

The stability of the Guangyuan mine dump was evaluated by using the strength parameters (cohesion *c* and internal friction angle ϕ) of the dump and weak bedding as uncertain variables. According to the physical and mechanical parameters of the computational model and the deformation characteristics of the dump, the parameter inversion analysis of the Guangyuan mine dump was carried out. That is, when the parameters were not reduced, the deformation characteristics of the model should be consistent with the results of the on-site survey. The numerical results show that the variation range of strength parameters affecting slope stability is shown in Table II, and the statistical characteristics of uncertain variables are counted in Table II according to formulas (15) and (16).

Uncertain variables	Cohesion of dump (kPa)	Internal friction angle of dump (°)	Cohesion of weak bedding (kPa)	Internal friction angle of weak bedding (°)
Range	[40, 60]	[26, 30]	[10, 30]	[12, 16]
Mean value	50	28	20	14
Mean square deviation	6.94	1.39	6.94	1.39

Table II Range of uncertain parameters and its statistical characteristics

4.3. STABILITY ANALYSIS OF GUANGYUAN MINE DUMP

The response surface function was assumed to be a quadratic polynomial function with no crossover term. It can be expressed as:

$$F_s-1=g(x)\approx G(x)=a+\sum_{i=1}^{n}b_ix_i+\sum_{i=1}^{n}c_ix_i^2 \quad (23)$$

where *a*, b_i, and c_i are the undetermined coefficients of the response surface function, and x_i is an uncertain variable, for *i*= 1, 2, 3, and 4.

The test points selected according to the Bucher test method are shown in Table III.

The strength reduction of each test point in Table III was calculated by numerical analysis and the safety factor of each test point was obtained. Then the stability function value of each test

Test points	Constant	Cohesion of dump (kPa)	Internal friction angle of dump (°)	Cohesion of weak bedding (kPa)	Internal friction angle of weak bedding (°)	$F_s - 1$
1	1	50	28	20	14	0.38
2	1	30	28	20	14	0.29
3	1	70	28	20	14	0.48
4	1	50	24	20	14	0.31
5	1	50	32	20	14	0.46
6	1	50	28	10	14	0.32
7	1	50	28	30	14	0.44
8	1	50	28	20	10	0.19
9	1	50	28	20	18	0.52

Table III The test points and the results of the test

point was found in the last column of Table III. According to the values of uncertain variable and response function of each test point, the response surface function of Guangyuan mine dump is as follows:

$$G(x) = -1.11 + 0.0035x_1 + 0.0012x_2 + 0.006x_3 + 0.083x_4 + 0.0003x_2^2 - 0.0016x_4^2 \quad (24)$$

When statistical characteristics of the uncertain variables in Table II are substituted into the formulas (20), (21), and (24), the mean value of the stability function is 0.3302, and the mean square deviation is 0.078. Therefore, the probability density function of the dump stability is as follows:

$$f(z) = \frac{1}{\sqrt{2\pi} \times 0.078} \exp\left[-(z-0.3302)^2 / \left(2 \times 0.078^2\right)\right] \quad (25)$$

Substituting formulas (25), (19) into formula (14), the instability probability P_f of Guangyuan dump is 0.087, which is less than the critical instability probability P_L=0.1599. So the stability of the Guangyuan mine dump is good. The evaluation results are consistent with the actual conditions of the mine dump.

After the sections being set up along the main slide direction at the No. 4 and No. 5 elevation platforms, the stability function was constructed based on the Swedish slice method, then the instability probability of this model was solved. The results are shown in Table IV, in which the FLAC+RSM method is the novel method determined in this study.

Method	Mean value	Mean square deviation	Instability probability	Safety factor
Sweden slice method	0.200	0.065	0.093	-
FLAC+RSM	0.330	0.078	0.087	-
FLAC^{3D} simulation	-	-	-	1.24

Table IV The results of different methods

According to the data in Table IV, the slope instability probability obtained by the Swedish slice method is larger than that by the novel method. This is because during the slope failure, No. 4 and No. 5 elevation platforms were destroyed before No. 3 elevation platform. However, the main sliding direction of No. 4 and No. 5 elevation platforms is adjacent to the spatial position of No. 3 elevation platform, which makes No. 3 elevation platform play a certain role in restraining the slippage damage of No. 4 and No. 5 elevation platforms. At the same time, the instability of the dump spread to No. 3 elevation platform, and then to No. 2 elevation platform, which resulted in the overall instability of the Guangyuan mine dump.

The stability function based on the novel method is obtained by simulating the real instability of the mine dump and it can reflect the deformation characteristics and interactions among different sliding bodies of the slope. However, the stability function based on the limit equilibrium method only represents the failure of single landslide of the selected section, and this cannot reflect the interactions among these platforms when Guangyuan mine dump is unstable as a whole. So the instability probability obtained by the limit equilibrium method is larger than the true value.

The safety factor of Guangyuan mine dump is 1.24 by using FLAC^{3D} technique. It is larger than 1.20 required by China Slope Engineering Specifications, that is the Guangyuan mine dump is stable. The results are consistent with the novel method described in this study. In addition, the similar mine dumps, such as Jinguanli mine dump, Antaibao mine dump, and Anjialing mine dump were evaluated by this method and all the evaluation results were consistent with the field monitoring data. Therefore, it is feasible to evaluate 3D slope stability by using the novel method and the results are more reliable than those based on the limit equilibrium method.

5. CONCLUSIONS

To give a definitive evaluation result by using the fuzzy probability method to evaluate 3D slope stability, renormalization group theory is introduced into the fuzzy probability analysis, and the slope stability analysis of the mine dump is carried out based on the novel method. The main conclusions are as follows:

(1) By introducing renormalization group theory into the fuzzy probability analysis, the critical instability probability of the slope is given and the subjectivity and fuzziness of evaluating the slope instability are weakened.

(2) When the fuzzy probability method is used to analyze the slope stability, the stability function is key to reflect the failure characteristics of the 3D slope engineering.

(3) The novel method has been verified by the practical 3D slope engineering. Compared with the stability function based on limit equilibrium method, the quantitative evaluation results are more effective and reliable.

The novel method can give a definitive evaluation result to evaluate the 3D slope stability, which can provide a reference for the similar engineering practice. However, to make this method more perfect, the energy transfer loss will be considered in the slope renormalization group model in the further study.

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