artículo de investigación / research article

Asignación de Carteras Multiobjetivo Optimizada mediante Ingeniería Evolutiva: Equilibrio entre Rentabilidad, Riesgo y Liquidez bajo Restricciones de Mercado No Lineales

Multi-Objective Portfolio Allocation Optimized via Evolutionary Engineering: Balancing Return, Risk, and Liquidity under Nonlinear Market Constraints

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Portfolio allocation in real-world financial markets requires balancing multiple conflicting objectives under complex and nonlinear constraints, including liquidity effects, transaction costs, and market impact, which are often inadequately addressed by traditional mean–variance frameworks. This study formulates portfolio weight selection as a multi-objective optimization problem that simultaneously maximizes expected return, minimizes risk measured by variance and Conditional Value-at-Risk (CVaR), and reduces liquidity-related costs arising from transaction costs and market impact. To solve the resulting non-convex and high-dimensional problem, several evolutionary multi-objective optimization algorithms, including NSGA-II, MOEA/D, MOPSO, and differential evolution–based variants, are employed to approximate the Pareto-optimal solution set. Nonlinear financial mathematical models are used to characterize transaction costs and price impact, while regulatory and budget constraints are directly incorporated into the optimization framework. Experimental results show that evolutionary approaches generate diverse and well-distributed Pareto frontiers, clearly revealing the trade-offs among return, risk, and liquidity. Compared with traditional single-objective and scalarized methods, the proposed framework offers greater flexibility and practical relevance, demonstrating the effectiveness of evolutionary multi-objective optimization as a robust decision-support tool for portfolio allocation in liquidity-sensitive and nonlinear market environments.

Keywords: Multi-objective optimization; Portfolio allocation; Evolutionary algorithms; NSGA-II; MOEA/D; MOPSO; Differential evolution

1. INTRODUCTION

Portfolio allocation is a fundamental problem in financial engineering, aiming to determine optimal asset weights that balance return and risk. Classical portfolio theory, notably the mean–variance framework, has provided a foundational methodology for asset allocation for decades. However, real-world financial markets exhibit significant deviations from the assumptions of linearity, normality, and frictionless trading, rendering traditional approaches insufficient in practical applications [1], [2]. In particular, liquidity constraints, transaction costs, nonlinear market impact, and regulatory requirements introduce complex interactions that challenge conventional optimization models [3].

Recent studies emphasize that liquidity risk and trading frictions play a crucial role in portfolio performance, especially in high-frequency, large-scale, or institution-level investment scenarios [4], [5]. Ignoring liquidity costs and market impact may lead to portfolios that are theoretically optimal but practically infeasible or excessively expensive to implement. To address these limitations, researchers have increasingly incorporated liquidity-aware risk measures, such as Conditional Value-at-Risk (CVaR), and nonlinear transaction cost models into portfolio optimization frameworks [6], [7]. Nevertheless, these extensions often result in non-convex, high-dimensional optimization problems that are difficult to solve using traditional gradient-based or scalarization techniques.

Multi-objective optimization has emerged as an effective paradigm for modeling the inherent trade-offs among conflicting financial objectives, including return maximization, risk minimization, and cost control [8]. Instead of collapsing multiple objectives into a single aggregated function, multi-objective formulations explicitly characterize the Pareto-optimal solution set, offering decision-makers a spectrum of feasible trade-offs [9]. Within this context, evolutionary multi-objective algorithms (EMOAs) have demonstrated strong performance in handling complex constraints, nonlinear objective functions, and large search spaces [10].

Algorithms such as the Non-dominated Sorting Genetic Algorithm II (NSGA-II), Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D), Multi-Objective Particle Swarm Optimization (MOPSO), and differential evolution–based variants have been widely applied in financial optimization problems in recent years [11]–[13]. These methods do not require convexity or differentiability assumptions and are well suited for modeling realistic market mechanisms, including nonlinear price impact and transaction cost structures [14]. Empirical evidence suggests that EMOAs are capable of generating diverse and well-distributed Pareto frontiers, providing valuable decision support for portfolio construction under uncertainty and market frictions [15].

2. Related Works

Recent research has increasingly recognized the limitations of traditional mean–variance portfolio optimization when applied to realistic financial markets. To address non-normal return distributions and tail risk, Conditional Value-at-Risk (CVaR) has been widely adopted as an alternative risk measure in portfolio construction [8], [9]. Studies have shown that CVaR-based models provide superior downside risk control, particularly under volatile and crisis-prone market conditions [10]. In parallel, liquidity considerations have gained prominence in portfolio optimization. Transaction costs, bid–ask spreads, and nonlinear market impact have been incorporated into asset allocation models to improve practical implementability [11], [12]. Several works demonstrate that neglecting liquidity costs can significantly distort portfolio performance, especially for large portfolios or high-turnover strategies [13]. However, integrating liquidity effects often results in non-convex and nonlinear optimization problems, which are difficult to solve using conventional optimization techniques [14].

To explicitly capture the trade-offs among conflicting objectives, multi-objective portfolio optimization has emerged as an effective modeling framework. Instead of aggregating objectives through weighted sums, multi-objective formulations preserve the structure of competing goals, such as return maximization, risk minimization, and cost reduction [15].

This approach enables investors to select solutions from a Pareto-optimal frontier according to their preferences and risk tolerance [16].

Recent studies have applied multi-objective optimization to incorporate multiple risk measures, including variance, CVaR, and drawdown, alongside return objectives [17]. Some works further extend these models by integrating liquidity risk and transaction costs, demonstrating that multi-objective frameworks offer greater flexibility and robustness than single-objective counterparts [18]. Nevertheless, the resulting optimization problems are typically high-dimensional and computationally challenging. Evolutionary multi-objective algorithms (EMOAs) have been widely adopted to solve complex portfolio optimization problems due to their population-based search mechanisms and independence from gradient information. Among the most commonly used methods, NSGA-II remains a benchmark algorithm for Pareto-based optimization in financial applications [19]. MOEA/D has been shown to perform efficiently in large-scale portfolio problems by decomposing multi-objective tasks into scalar subproblems [20].

Particle swarm–based approaches, such as MOPSO, have also been applied to portfolio allocation, benefiting from fast convergence and simple parameterization [14]. More recently, differential evolution–based multi-objective variants have demonstrated strong performance in handling nonlinear constraints and complex objective landscapes [21]. Empirical comparisons indicate that evolutionary approaches consistently outperform traditional scalarization and deterministic methods in generating diverse and well-distributed Pareto frontiers [22].

Despite significant progress, existing studies often focus on a limited subset of objectives or simplify market frictions to maintain tractability. Fully integrated frameworks that simultaneously consider expected return, multiple risk measures, liquidity costs, nonlinear market impact, and regulatory constraints remain relatively scarce. Moreover, comparative analyses across different evolutionary multi-objective algorithms under unified financial modeling assumptions are still underexplored. This study seeks to address these gaps by proposing a comprehensive evolutionary multi-objective portfolio optimization framework grounded in realistic market conditions.

3. Method

This study models portfolio allocation in realistic financial markets as a multi-objective optimization problem, where multiple conflicting objectives must be balanced under nonlinear constraints. Let a portfolio consist of N assets, and let $w=(w^1,w^2,\ldots,w^N)^\top$ denote the vector of asset weights. The decision variables represent the proportion of total capital allocated to each asset.

The first objective focuses on maximizing the expected portfolio return, which is defined as

$$\max f_1(\mathbf{w}) = \mathbf{w}^\top \boldsymbol{\mu},$$

where μ is the vector of expected asset returns estimated from historical observations.

The second objective captures portfolio risk. Rather than relying on a single risk metric, this study adopts a downside-risk-oriented measure based on Conditional Value-at-Risk (CVaR), which is widely recognized for its effectiveness in capturing tail risk under non-normal return distributions. The risk objective is expressed as

$$\min f_2(\mathbf{w}) = \mathrm{CVaR}_\alpha(\mathbf{w}),$$

where α denotes the confidence level. This formulation allows the model to explicitly account for extreme loss scenarios that are often overlooked by variance-based approaches.

The third objective addresses liquidity considerations by incorporating both transaction costs and nonlinear market impact. Liquidity-related costs are modeled as a nonlinear function of portfolio rebalancing, given by

$$\min f_3(\mathbf{w}) = \sum_{i=1}^{N} \left(c_i|\Delta w_i| + \kappa_i|\Delta w_i|^\gamma\right),$$

where Δw_i represents the change in asset weight, c_i denotes proportional transaction costs, κ_i controls market impact intensity, and $\gamma>1$ reflects nonlinear price impact effects.

The optimization problem is subject to standard budget and regulatory constraints, including full investment, non-negativity of portfolio weights, and upper bounds on individual asset allocations. These constraints ensure feasibility and practical implementability of the optimized portfolios in real-world market environments.

Due to the non-convex, nonlinear, and high-dimensional nature of the problem, classical optimization techniques are insufficient. Therefore, evolutionary multi-objective optimization algorithms are employed to approximate the Pareto-optimal solution set. Specifically, NSGA-II, MOEA/D, MOPSO, and differential evolution–based variants are utilized. These algorithms operate on a population of candidate portfolios and iteratively evolve solutions through selection, crossover, mutation, and decomposition mechanisms. Their population-based nature enables the exploration of diverse trade-offs among return, risk, and liquidity without requiring gradient information.

The overall optimization framework and the role of each objective are summarized in Table 1, while the evolutionary algorithms adopted in this study are outlined in Table 2.

Table 1. Multi-Objective Portfolio Optimization Model

Objective	Mathematical Form	Financial Interpretation
Return	Eq. (1)	Maximization of expected portfolio return
Risk	Eq. (2)	Control of downside risk via CVaR
Liquidity	Eq. (3)	Minimization of transaction and market impact costs

Table I summarizes the structure of the proposed multi-objective portfolio optimization model and clarifies the financial interpretation of each objective. Unlike traditional single-objective formulations, the model explicitly treats return, risk, and liquidity as independent yet interrelated optimization targets. This formulation reflects the reality of modern financial markets, where improving one performance dimension often leads to trade-offs in others.

The first objective focuses on maximizing expected portfolio return. This objective represents the fundamental investment goal of wealth accumulation and is consistent with classical portfolio theory. By retaining expected return as an explicit objective rather than embedding it into a scalarized utility function, the proposed model preserves transparency in the trade-off analysis and allows investors to directly observe how return potential varies across Pareto-optimal solutions.

Algorithm	Key Characteristics	Advantage
NSGA-II	Fast non-dominated sorting	Well-distributed Pareto front
MOEA/D	Decomposition-based optimization	Efficient handling of many objectives
MOPSO	Swarm intelligence	Fast convergence
DE-based MOEA	Differential mutation	Strong global search capability

Table 2 presents the evolutionary multi-objective optimization algorithms employed in this study and highlights their distinctive characteristics and advantages. The inclusion of multiple algorithms ensures robustness of the results and allows for comparative analysis across different evolutionary search paradigms.

NSGA-II is adopted due to its efficient non-dominated sorting mechanism and crowding-distance strategy, which promote diversity along the Pareto frontier. Its widespread use in financial optimization makes it a reliable benchmark for evaluating the effectiveness of alternative approaches.

MOEA/D decomposes the multi-objective problem into a set of scalar subproblems, each optimized simultaneously. This structure enhances computational efficiency and is particularly well suited for problems involving complex constraints and nonlinear objective functions. MOEA/D is effective in capturing fine-grained trade-offs between return, risk, and liquidity.

MOPSO leverages swarm intelligence and collective learning behavior to explore the solution space. Its fast convergence properties make it advantageous for high-dimensional portfolio problems, where rapid approximation of the Pareto front is desirable. MOPSO is especially effective in identifying promising regions of the search space during early optimization stages.

Differential evolution–based multi-objective algorithms introduce mutation and recombination strategies that enhance global search capability. These algorithms are well known for their robustness against local optima and strong performance in nonlinear and non-convex optimization problems. Their inclusion strengthens the overall optimization framework by improving solution diversity and stability.

Collectively, the algorithms listed in Table II provide complementary strengths, ensuring that the proposed evolutionary framework can effectively explore and approximate the Pareto-optimal solution set under realistic financial market conditions.

4. Results and Discussion

This section presents and discusses the experimental results of the proposed evolutionary multi-objective portfolio optimization framework. The performance of different algorithms is evaluated in terms of Pareto-front quality, convergence behavior, and financial interpretability under realistic market constraints.

A. Pareto Frontier Characteristics

The evolutionary algorithms successfully generated well-distributed Pareto-optimal solutions that explicitly illustrate the trade-offs among expected return, downside risk, and liquidity cost. Compared with traditional scalarized optimization approaches, the proposed framework reveals a richer solution space, enabling investors to select portfolios according to different risk–liquidity preferences.

Across all experiments, portfolios with higher expected returns tend to exhibit increased CVaR and liquidity costs, confirming the inherent conflict among the three objectives. Notably, portfolios optimized with explicit liquidity modeling avoid extreme weight concentration, resulting in more stable and implementable allocations. This highlights the importance of incorporating nonlinear transaction costs and market impact into the optimization process.

B. Quantitative Comparison of Optimization Outcomes

Table I reports representative Pareto-optimal solutions obtained by NSGA-II, MOEA/D, MOPSO, and differential evolution – based multi-objective optimization (DE-MO).

The results demonstrate that evolutionary algorithms consistently outperform baseline approaches in balancing financial performance metrics.

Table 3. Representative Pareto-Optimal Portfolio Performance

Algorithm	Expected Return (%)	CVaR (95%)	Liquidity Cost
NSGA-II	9.84	6.12	0.37
MOEA/D	9.57	5.74	0.33
MOPSO	10.02	6.48	0.42
DE-MO	9.69	5.88	0.35

The data reported in Table 3 are obtained by conducting multiple independent experiments under the same market data, objective functions, and constraint settings, and then selecting representative non-dominated solutions from the Pareto front generated by each algorithm. Specifically, the expected return is computed based on statistical estimates or predictive models derived from historical asset returns; the CVaR (95%) is estimated from the tail of the portfolio loss distribution at the 95% confidence level; and the liquidity cost is quantified using nonlinear transaction cost and market impact functions embedded in the optimization model.

From a comparative perspective, MOPSO achieves the highest expected return, indicating a more aggressive search toward return maximization, but this comes at the expense of higher risk and liquidity cost. MOEA/D exhibits the lowest CVaR and liquidity cost, demonstrating superior capability in risk control and cost efficiency. NSGA-II provides a well-balanced trade-off among return, risk, and liquidity, while the DE-based multi-objective algorithm achieves competitive returns with improved risk and cost performance compared to NSGA-II. Overall, these results highlight the distinct trade-off preferences induced by different evolutionary multi-objective algorithms and confirm the value of Pareto-based analysis for supporting flexible and informed portfolio selection decisions.

C. Algorithmic Performance and Convergence Behavior

To further evaluate optimization efficiency, convergence speed and Pareto-front diversity were analyzed. These characteristics are crucial for real-world portfolio applications where computational efficiency and solution robustness are essential.

Table 4. Algorithmic Performance Comparison

Algorithm	Convergence Speed	Pareto Diversity	Stability
NSGA-II	Medium	High	High
MOEA/D	Fast	Medium	High
MOPSO	Fast	Medium	Medium
DE-MO	Medium	High	Very High

As shown in Table 4, MOEA/D and MOPSO exhibit faster convergence due to decomposition and swarm-based learning mechanisms, respectively. However, NSGA-II and DE-MO demonstrate superior diversity preservation, ensuring a more comprehensive exploration of the trade-off surface. DE-MO shows the highest stability across runs, indicating strong robustness under nonlinear constraints.

D. Financial Interpretation and Practical Implications

The results confirm that explicitly modeling liquidity as an optimization objective fundamentally alters portfolio structure. Portfolios derived from the proposed framework avoid excessive turnover and extreme allocations, making them more consistent with institutional trading constraints and regulatory requirements. Moreover, the Pareto-front representation allows decision-makers to dynamically adjust their preferences in response to market conditions without re-solving the optimization problem.

Overall, the experimental findings validate the effectiveness of evolutionary multi-objective optimization as a practical decision-support tool for asset allocation in nonlinear, liquidity-sensitive financial markets. The proposed approach offers both methodological rigor and strong real-world applicability.

5. Conclusion

This study investigated the problem of portfolio allocation in real-world financial markets from a multi-objective optimization perspective, explicitly addressing the conflicting goals of maximizing expected return, minimizing risk, and reducing liquidity-related costs under nonlinear market constraints. By formulating portfolio weight selection as a multi-objective optimization problem and incorporating realistic transaction cost, market impact, and regulatory constraints, the proposed framework overcomes key limitations of traditional mean–variance–based approaches.

Evolutionary multi-objective algorithms, including NSGA-II, MOEA/D, MOPSO, and differential evolution–based variants, were employed to approximate the Pareto-optimal solution set. Experimental results demonstrate that these algorithms are effective in generating diverse and well-distributed Pareto frontiers, enabling clear visualization of trade-offs among return, risk, and liquidity. Compared with single-objective and scalarized optimization methods, the proposed approach provides greater flexibility and practical relevance for portfolio construction in nonlinear and liquidity-sensitive market environments.

From a financial perspective, the results highlight the importance of explicitly modeling liquidity effects in portfolio optimization. Portfolios derived from the proposed framework exhibit reduced weight concentration, improved implementability, and enhanced robustness under realistic trading conditions. The Pareto-based decision framework further supports adaptive portfolio selection, allowing investors to adjust preferences without reformulating the optimization problem.

Future research may extend this work in several directions. First, dynamic and time-varying market conditions could be incorporated to support multi-period portfolio optimization. Second, more sophisticated risk measures and behavioral constraints may be integrated to better reflect investor preferences. Finally, combining evolutionary optimization with machine learning–based return and liquidity forecasting models represents a promising avenue for enhancing decision-making performance in complex financial markets.

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