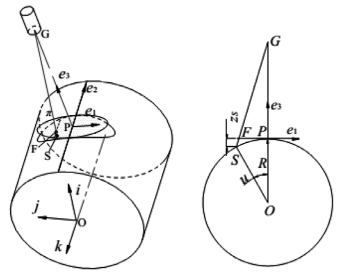
# Computation of spraying deposition thickness on parametrized surfaces using moving frame method



## CÁLCULO DEL ESPESOR DE DEPOSICIÓN POR PULVERIZACIÓN EN SUPERFICIES PARAMETRIZADAS, UTILIZANDO EL MÉTODO DE MARCO MÓVIL

DOI: http://dx.doi.org/10.6036/7493 | Recibido: 04/12/2014 • Aceptado: 27/01/2015

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## RESUMEN

- Este documento analiza las dificultades y las limitaciones actuales de los enfoques que utilizan coordenadas de referencia estáticas en la planificación de trayectorias adecuadas para las pistolas de pulverización robotizadas, en su intento de aplicar incluso espesores de revestimiento a superficies curvas. Ahora se propone un método de cálculo, utilizando un marco móvil como método, que se puede utilizar para modelizar con éxito tales trayectorias para la pulverización continua en superficies parametrizadas. Se definen dos coeficientes. El primero es la relación entre la velocidad de acumulación de espesor de revestimiento en cualquier punto de la superficie pulverizada en comparación con la del centro de pulverización, el otro relaciona la posición y la orientación en cualquier punto relativo a la pistola de pulverizado. Tomando una superficie de rotación, por ejemplo, y teniendo en cuenta la velocidad de movimiento de la pistola y el espacio de pulverizado como variables, aplicamos el método para calcular  el espesor de deposición de la pulverización continua, con el fin de conseguir una trayectoria optimizada para minimizar la desviación en el espesor de recubrimiento de la superficie pulverizada. También aplicamos el programa Matlab para llevar a cabo una verificación de los cálculos numéricos, el resultado del cual demuestra que este método informático representa una mejora con respecto a métodos estáticamente programados tanto en términos de aplicabilidad como de validez, para la planificación de trayectorias de pulverizado en superficies parametrizadas.
- **Palabras clave:** Espesor del recubrimiento pulverizado, marco móvil, ratio de velocidad de pulverización, posición de pulverización y coeficiente de orientación.

## ABSTRACT

This paper discusses the difficulties and current limitations of approaches which use static reference coordinates when planning appropriate trajectories for robotic spray guns as they attempt to apply even coating thicknesses to curved surfaces. It then proposes a calculative method, using moving frame of method, which can be used for successfully modeling such trajectories for continuous spraying onto parametric surfaces. Two coefficients are defined. The first is the ratio of the velocity of accumulation of coating thickness at any point in the sprayed area compared with that the spraying center, another one relates the position and orientation at any point relative to the spray gun. Taking a surface of rotation as an example, and considering the travel velocity of the spray gun and the spraying space as variables, we apply the method to compute the continuous-spraying deposition thickness, in order to allow trajectory optimization to minimize coating thickness deviation on the sprayed surface. We also apply Matlab programming to conduct a numerical computation verification, the result of which shows that this computing method represents an improvement over static coordinate methods in terms both of applicability and validity for spray-trajectory planning for the parametrized surfaces.

**Keywords:** Spray Coating Thickness, Moving Frame, Spraying Velocity Ratio, Spraying Position and Orientation coefficient**.**

## 1. INTRODUCTION

Spraying robots are widely used in industrial automation, where uniformity of coating thickness on products is mainly dependent on appropriate planning of the spray gun trajectory and establishing an effective and practical paint deposition model for robotic spray. Currently sprayed surfaces model consists of the mesh model and the parametric CAD model. Chen [1-3] proposed to determine spray trajectory basing on the mesh model. In this method, the surface is approximated into a number of small triangular facets, and the adjacent small planes with  same or similar normal vectors are merged into a large patches to ensure the final patches to be relatively flat and to simplify the spray trajectory planning for free-form surface. The spray trajectory of the entire surface is then determined by connecting the spray trajectories for each of the large planes. BI [4] used a similar method. However BI directly defined the spray cone through specifying markers of sprayed, and trajectory planning was regarded as the connection of these markers. Thus the planning problem becomes a traveling salesman problem. Zhao and Chen [5-6] optimized the robot paint trajectories for complex free-form surfaces based on techniques of point cloud slicing and a genetic algorithm. However, such mesh methods present the difficulty [3] when planning connection of short paint trajectories over small surfaces. The method of trajectory planning based on parametric CAD model, essentially approximates complex surfaces to a simple, regular surface. Prasad [7-8] hierarchically segmented complex automotive surface into geometrically as well as topologically simple extruded surfaces. The least square method was used to fit big curvature surfaces as natural quadric shapes, such as cylindrical, conical and spherical surfaces [9-10], however, such approximated generation of trajectory can cause large deviations in coating thickness while complex surfaces cannot be approximated in terms of simple surfaces. There is little literature regarding of spray coordinate while planning trajectory of spray gun, others only set that the direction of the spray gun (the $z$-direction) should be the normal vector to the surface at the point being sprayed. Neither is there a discussion of the relationship between the trajectory of the spray gun and the shape, metric of the sprayed surface.

The model of coating thickness usually is established under the local coordinate system of the center of the sprayed area as it moves along with the spray gun, and a difficulty of spray trajectory planning to obtain uniformity of paint thickness is mainly related to the shape of the surface being sprayed. The primary method for studying the surface and curve characteristics is differential geometry, while the method of moving frame mainly is used in the geometry of space. The core concept of which is to study the geometric invariants related to the transformation of the frame, the obtained results will therefore not be influenced by differences of the coordinate systems. This paper provides a new method for calculating deposition thickness on a parametric surface during continuous spraying, and studies the relationships among the model of paint thickness, spray trajectory, surface shape and metric by means of the geometric characteristics of the surface and differential geometry theory. The continuous deposition thickness on the surface is calculated on the basis of the method of moving frame. We also consider the case of using surfaces of revolution to calculate the paint thickness resulting from a continuous trajectory, and to provide a means of determining appropriate spray trajectories for such surfaces of revolution.

## 2. COMPUTATION OF SPRAYING DEPOSITION ON A SURFACE

### 2.1 MODEL OF SPRAYING DEPOSITION ON A PLANE

At present, a number of scholars have studied applying different deposition models of trajectory planning and their effects on the painting deposition thickness, these include Gaussian distribution profiles [11], β distribution profile [12], as well as more complex and asymmetrical distribution models [13]. These investigations show that β distribution is the preferred model. On the basis of the experiment, Zhang Y.G. [14] proposed an elliptic double β distribution model which reflects the distribution of film thickness within the elliptical spray area formed using an air gun in the plane of the work piece. The film thickness distribution function of any point ($x$, $y$) in the elliptical spray area in a unit time is:

$$q(x,y)=q_u\left(1-\frac{x^2}{a^2}\right)^{\beta_1-1}\left(1-\frac{y^2}{b^2\left(1-x^2/a^2\right)}\right)^{\beta_2-1}, \quad (1)$$

$$-a\le x\le a, -b\left(1-x^2/a^2\right)^{1/2}\le y\le b\left(1-x^2/a^2\right)^{1/2}$$

where $a$, $b$ is respectively the half of long and short axes of ellipse area, and $q_u$ represents the maximum value of film thickness in that area.

### 2.2 COMPUTATION OF SPRAYING DEPOSITION ON A CURVED SURFACE

The deposition model from an air spray gun onto a curved surface is showed as Fig.1. The thickness distribution proposed by Chen [1] is considered as the projection onto the actual sprayed surface of the sprayed material from the imaginary spraying plane $\pi$, which is expressed as equation 2:

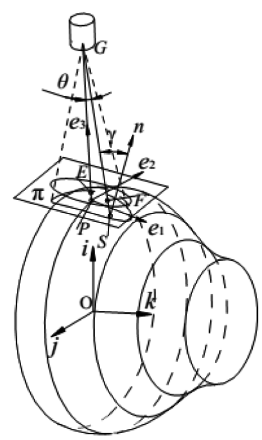



*Fig.1: Spray model of a surface*

$$q_S=q_F\left(\frac{h}{l}\right)^2\frac{\cos\gamma}{\cos^3\theta} \quad (2)$$

where, $l$ is the distance between the center point $\boldsymbol{G}$ of spray gun and a point $\boldsymbol{S}$ on the actual sprayed surface, $h$ is the distance between the center point $\boldsymbol{G}$ of spray gun and imaginary spraying plane $\pi$, while $q_S$ and $q_F$ respectively represent the thickness of the spray coating at point $\boldsymbol{S}$ of the sprayed surface and at point $\boldsymbol{F}$ on the imaginary spraying plane $\pi$. The equation for calculating $q_F$ is Eq.1, and γ represents the angle between normal vector $\boldsymbol{n}$ and the spray-ray $GS$ at point $\boldsymbol{S}$ of the surface, and θ represents the angle of spraying cone, point $\boldsymbol{E}$ is the central point of the elliptical area of the spraying plane $\pi$, and point $\boldsymbol{P}$ is a point of intersection of the spray gun center axis and the actual sprayed surface. The motion trail of point $\boldsymbol{P}$ is the spray trajectory.

Wei[15]discussed applying the method of the curvature circle to calculate the angle γ in Eq.2, but did not explain which curvature circle in the osculating plane should be, the radius of curvature can be obtained intuitively using a cylinder surface as an example. In the case of non-extruded surfaces, although the curvature circle of the normal section curve may be determined in this method, the process is very complicated. Meanwhile, Wei [15] regarded that the curvature circle method was more practical for a free-form surface represented by triangulated mesh, and Taubin[16] method was the most common calculation method for discrete curvature. However we discovered that the curvature circle is a circle on the normal section at point $\boldsymbol{S}$, which is determined by line $GS$ and normal vector of point $\boldsymbol{S}$ on the surface, here the curvature should be a normal curvature, but in Taubin method used by Wei[15] was a main curvature. This paper shows how angle $\gamma$ is determined by calculating the normal vector $\boldsymbol{n}_S$ at point $\boldsymbol{S}$ on the surface, based on the method of moving frame.

As is shown in Fig.1, the coordinates $\{\boldsymbol{O};\ \boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{k}\}$ is the reference coordinate system for the surface. We can establish a moving frame of reference $\{\boldsymbol{P};\ \boldsymbol{e}_1,\ \boldsymbol{e}_2,\ \boldsymbol{e}_3\}$ at point $\boldsymbol{P}$ along the spray trajectory of the surface. Point $\boldsymbol{P}$ is a point in the spray trajectory, and point $\boldsymbol{E}$ is a point of intersection of the spray gun axis and the spraying plane $\pi$. In order to simplify calculation, supposes the spray plane $\pi$ to be a tangential plane at point $\boldsymbol{P}$ on the surface, when point $\boldsymbol{E}$ coincides with point $\boldsymbol{P}$, point $\boldsymbol{F}$ is the intersection point of the line GS and the spray plane $\pi$. The vector $\boldsymbol{e}_2$ is the tangential direction along the spray trajectory at point $\boldsymbol{P}$, and $\boldsymbol{e}_3$ is the normal vector of the surface at this same point, $\boldsymbol{e}_1 = \boldsymbol{e}_2 \times \boldsymbol{e}_3$, and we can obtain $\cos\gamma$ and $\cos\theta$ under the moving frame of reference $\{\boldsymbol{P};\ \boldsymbol{e}_1,\ \boldsymbol{e}_2,\ \boldsymbol{e}_3\}$:

$$\cos\gamma = \frac{\boldsymbol{n}_S \cdot \overrightarrow{SG}}{|\boldsymbol{n}_S| \cdot |\overrightarrow{SG}|} = \frac{\boldsymbol{n}_S \cdot \overrightarrow{SG}}{\sqrt{g_{11}g_{22} - g_{12}^2} \cdot l},$$

$$\cos\theta = \frac{\overrightarrow{GP} \cdot \overrightarrow{GS}}{|\overrightarrow{GP}| \cdot |\overrightarrow{GS}|} = \frac{(0,0,h)\cdot(-x_S, -y_S, h - z_S)}{h \cdot l} = \frac{h - z_S}{l}$$

Then, Eq.2 can be written as:

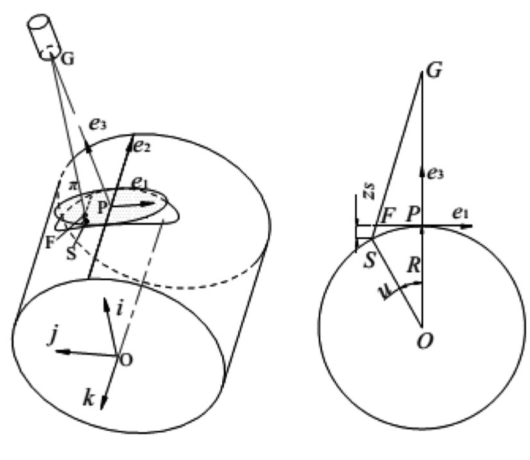
$$q_S = q_F \frac{h^2}{(h - z_S)^3} \frac{(\boldsymbol{n}_S \cdot \overrightarrow{SG})}{|\boldsymbol{n}_S|} = q_F c_S\ , \quad c_S = \frac{h^2}{(h - z_S)^3} \frac{(\boldsymbol{n}_S \cdot \overrightarrow{SG})}{|\boldsymbol{n}_S|} \qquad (3)$$

where $z_S$ is the coordinate component of point $\boldsymbol{S}$ along vector $\boldsymbol{e}_3$ under the moving frame of reference $\{\boldsymbol{P};\ \boldsymbol{e}_1,\ \boldsymbol{e}_2,\ \boldsymbol{e}_3\}$, and $|\boldsymbol{n}_S|$ is the length of the normal vector at that point, which can be expressed as $|\boldsymbol{n}_S| = \sqrt{g_{11}g_{22} - g_{12}^2}$, and $g_{11}$, $g_{22}$, $g_{12}$ are the first fundamental invariants of the surface at point $\boldsymbol{S}$.

Definition-1: The parameter $c_S$ determined by Eq.3, describes the position, orientation and metric of the surface at that point with respect to the axis of the spray gun, which is called the spraying coefficient of orientation of point $\boldsymbol{S}$.

It can be seen from Eq.3 that the parameter $z_S$ is expressed in the 'moving' coordinate system, while the normal vector $\boldsymbol{n}_S$ is usually expressed in the 'static' reference coordinate system of the surface. Therefore, $\frac{h^2}{(h-z_S)^3}$ is calculated under the moving frame, while $\frac{(\boldsymbol{n}_S \cdot \overrightarrow{SG})}{\boldsymbol{n}_S}$ is calculated under the reference coordinate system of the surface or the moving frame of reference. Therefore the computation of the spray thickness at point $\boldsymbol{S}$ on the surface includes two components: $q_F$ and $c_S$, for any given spraying condition, that is, when the parameters $a$, $b$, $q_u$, $\beta_1$ and $\beta_2$ in Eq.1 are given, $q_F$ is related only to the $x$ and $y$ values in the moving frame of reference and can be calculated under the frame of reference. Therefore, the deposition thickness at any point on the surface is not only related to the $x$ and $y$ values of point $\boldsymbol{S}$ in the area of spray plane $\pi$, but also related to the height $h$ of the spray gun, the coordinate value $z_S$ of this point, also the normal vector $\boldsymbol{n}_S$ of the point on the surface and the first fundamental invariants. Then using Eq.3 and definition-1 make the computation of spraying deposition thickness at any point on the surface simple and effective.

In order to compare the computation method employed in this paper with that in literature[15], we used the same parameters: $a$=142.112mm, $b$=43.032mm, $q_{max}$=81.12μm, $\beta_1$ = 4.1987, $\beta_2$=6.1692, and the cylinder diameter of the sprayed cylinder surface Φ= 275mm. after substituting the above parameters into Eq.2-Eq.3 proposed in our paper, the numerical simulation result of the corresponding point's spraying deposition thickness of the sprayed cylinder surface when $x_F$=-43.5mm were obtained. Further more, the coordinate values of the corresponding point $F(x_F, y_F, z_F)$ of the imaginary spraying plane and that of the corresponding point $S(x_S, y_S, z_S)$ of the sprayed cylinder surface when $x_F$=-43.5mm were calculated, the results are listed in Table I below. As can be seen from it, the coefficients of spraying orientation $c_S$ of the sprayed points with same X value are equal, and are the coordinate values of $z_S$, the results also can be seen from Fig.2 (b). But the results of $z_S$ given by the literature [15] were not equal when using the same X value.




(a) 3D drawing (b) 2D projection

*Fig. 2: Spray paint deposition model of cylinder*

## 3. CONTINUOUS SPRAYING DEPOSITION THICKNESS FOR A PARAMETRIZED SURFACES

### 3.1 DEPOSITION MODEL FOR CONTINUOUS SPRAYING ALONG CURVED TRAJECTORIES

When the spray gun moves at a constant speed along a curved trajectory, the accumulated thickness $q_S$ at a point $\boldsymbol{S}$ on the surface can be expressed as:

$$q_S(x_F) = \int_0^t q_F c_S dt\ , \quad t = \frac{2b\left(1 - x_F^2/a^2\right)^{1/2}}{v} \qquad (4)$$

| Sequence number | Coordinates of the point $F(x_F, y_F, z_F)$ under the moving frame (mm) | Coordinates of the corresponding point $S(x_S, y_S, z_S)$ under the moving frame (mm) | Coefficient of spraying orientation $c_S$ | Theoretical value of paint thickness (μm) |
|---|---|---|---|---|
| 1 | (-43.5, -30, 0) | ( -44.9970,-31.0324, -7.5711) | 0.8226 | 0.7103 |
| 2 | (-43.5, -25, 0) | ( -44.9970,-25.8604, -7.5711) | 0.8226 | 3.6200 |
| 3 | (-43.5, -20, 0) | ( -44.9970,-20.6883, -7.5711) | 0.8226 | 10.2297 |
| 4 | (-43.5, -15, 0) | ( -44.9970,-15.5162, -7.5711) | 0.8226 | 20.3415 |
| 5 | (-43.5, -10, 0) | ( -44.9970,10.3441, -7.5711) | 0.8226 | 31.6361 |
| 6 | (-43.5, -5, 0) | ( -44.9970,-5.1721, -7.5711) | 0.8226 | 40.5549 |
| 7 | (-43.5, 0, 0) | ( -44.9970,0, -7.5711) | 0.8226 | 43.9445 |
| 8 | (-43.5, 5, 0) | ( -44.9970,5.1721, -7.5711) | 0.8226 | 40.5549 |
| 9 | (-43.5, 10, 0) | ( -44.9970,10.3441, -7.5711) | 0.8226 | 31.6361 |
| 10 | (-43.5, 15, 0) | ( -44.9970,15.5162, -7.5711) | 0.8226 | 20.3415 |
| 11 | (-43.5, 20, 0) | ( -44.9970,20.6883, -7.5711) | 0.8226 | 10.2297 |
| 12 | (-43.5, 25, 0) | ( -44.9970,25.8604, -7.5711) | 0.8226 | 3.6200 |
| 13 | (-43.5, 30, 0) | ( -44.9970,31.0324, -7.5711) | 0.8226 | 0.7103 |

*Table I : Verification of spray painting deposition model of cylinder*

where $v$ is the spray gun velocity at the center point $\boldsymbol{P}$ in the spray ellipse, and $q_F$ is the coating thickness at any given point $x = x_F$ within the elliptical area of the spray plane and can be expressed as:

$$q_F = q_u\left(1-\frac{x_F^2}{a^2}\right)^{\beta_1-1}\left(1-\frac{\left(b\left(1-\frac{x_F^2}{a^2}\right)^{1/2}-vt\frac{\rho_F}{\rho_P}\right)^2}{b^2\left(1-\frac{x_F^2}{a^2}\right)}\right)^{\beta_2-1} \tag{5}$$

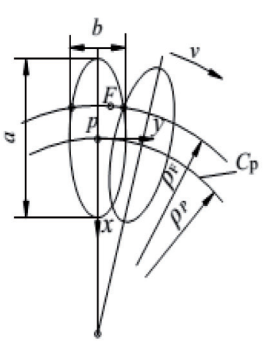
As is shown in Fig.3 (a), for the curved trajectory of a plane, here $\rho_F$ and $\rho_P$ are the relative radii of curvature at point $\boldsymbol{F}$ on the spray plane area, at point $\boldsymbol{P}$ of the spray center respectively.

Definition-2: The parameter $\eta_F$ represents the ratio of the accumulation velocity of coating thickness at the center and any point within the spray region along the spray trajectory. It called the speed ratio coefficient, which can be expressed as:

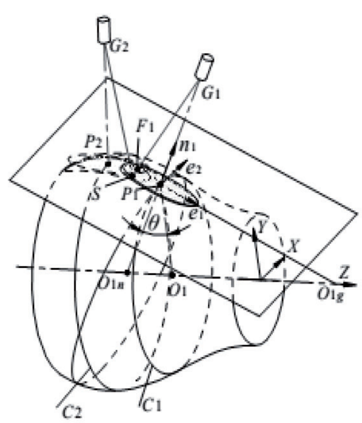
$$\eta_F = \frac{\rho_F}{\rho_P} \tag{6}$$

Supposes the curvature of the spray trajectory curve $Cp$ on the surface is $k$, then the normal curvature is $k_n = k\cos\varphi$, and the radius of curvature is $\rho_F = \frac{1}{k_n} = \frac{1}{k\cos\varphi}$, where, $\phi$ is the angle between the principal normal vector of the curve and the normal vector of the surface.

As for the curved trajectory on a curved surface, which is as shown in Fig.3 (b), $C_1$ and $C_2$ are the spray trajectories of two latitude circles on the surface of revolution, $G_1$ and $G_2$ are the spray gun positions on the their trajectories, and $P_1$ and $P_2$ are the points of intersection of the spray gun axes and the surface, namely the centers of the elliptical spray regions. Generally, the spray trajectories $C_1$ and $C_2$ are not coincident with geodesics on the surface, and so the rates of coating ac-




*(a) Continuous spraying along curved trajectory in a plane*




*(b) Continuous spraying in a curved surface*

*Fig.3: Spraying deposition thickness model of a curved trajectory*

cumulation on either side of the trajectory of the spray gun are not the same. In fact, $v_1/v_{F1}=\rho_{g1}/\rho_{gF1}$, where, $\rho_{g1}$ and $\rho_{gF1}$ are the radii of the geodesic curvatures of the trajectories corresponding to points $P_1$ and $F_1$ in the surfaces formed by spraying ray and the surface intersecting. By differential geometry we have $k_g=k\sin\theta$, where $k_g$ is the geodesic curvature of the surface, $\theta$ is the angle between the principal normal vector of the curve and the normal vector of the surface. As shown in Fig.3 (b), we have $\rho_g=\rho/\sin\theta$, therefore the speed ratio coefficient on the curved surface is:

$$\eta_F=\frac{\rho_{gF}}{\rho_{gP}}=\frac{\rho_{gP}-x_F}{\rho_{gP}}=1-\frac{x_F\sin\theta}{\rho_P} \tag{7}$$

where, $x_F$ is the coordinate component of point $\boldsymbol{F}$ along the direction of the long axis within the elliptical spray region. It can be seen from Eq.4-Eq.7 that the model of the accumulation thickness from continuous spraying along a trajectory is not only related to the spraying deposition thickness in a unit time at the sprayed point, the spray gun velocity, but also secondary fundamental invariant of the surface, such as, the radius of the geodesic curvature of spray center and the sprayed point along the trajectory, that is, related to the shape of the surface. It is therefore necessary to derive the relationship between the surface parameters and the coordinate component $x_F$ in the moving frame of reference to compute.

The vector of point $\boldsymbol{S}$ of the surface under the reference coordinate system $\{\boldsymbol{O};\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$ is expressed as $\overrightarrow{OS}=\mathbf{T}_P^O\overrightarrow{PS}$, where the transformation matrix $\mathbf{T}_P^O$ is an expression of the orthogonal moving frame of reference $\{\boldsymbol{P};\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3\}$ under the surface reference coordinate system $\{\boldsymbol{O};\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$, so that

$$\overrightarrow{PS}=\left(\mathbf{T}_P^O\right)^{-1}\overrightarrow{OS} \tag{8}$$

If $(\tilde{x},\tilde{y},\tilde{z})$ are used to represent the coordinate values of any point of the surface under the reference coordinate system $\{\boldsymbol{O};\boldsymbol{i},\boldsymbol{j},\boldsymbol{k}\}$, and $(x, y, z)$ are used to represent the coordinate component of that point under the moving frame of reference $\{\boldsymbol{P};\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3\}$ at point $\boldsymbol{P}$, then, any given point $\boldsymbol{S}$ $(\tilde{x}_S,\tilde{y}_S,\tilde{z}_S)$ can be represented as the following expression under the moving frame of reference $\{\boldsymbol{P};\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3\}$

$$\begin{pmatrix}x_S & y_S & z_S & 1\end{pmatrix}^{\mathrm{T}}=\left(\mathbf{T}_P^O\right)^{-1}\begin{pmatrix}\tilde{x}_S & \tilde{y}_S & \tilde{z}_S & 1\end{pmatrix}^{\mathrm{T}} \tag{9}$$

Taking an air spray gun as an example, suppose that the spraying rays from the spray gun are straight lines, and that the atomization is a circular cone with the nozzle as its vertex. As is shown in Fig.1, the following relationship under the moving frame of reference can be derived:

$$\begin{cases}x_F=\dfrac{h}{h-z_S}x_S\\[2mm] y_F=\dfrac{h}{h-z_S}y_S\end{cases} \tag{10}$$

where, $x_F$ and $y_F$ are the coordinates of point $\boldsymbol{F}$ on the elliptical spray plane under the moving frame of point $\boldsymbol{P}$, then parameters $x_S$, $y_S$, $x_F$ and $c_S$ can be calculated using Eq.9 and Eq.10.

## 3.2 COMPUTATION OF CONTINUOUS SPRAYING DEPOSITION THICKNESS ON A SURFACE OF REVOLUTION

Surface modeling with rotation is a method commonly used in three-dimensional product. The surface of revolution is a regular surface which there sometimes exist local large curvature. Theoretical and experimental results indicated [7] that when the spray trajectory over the surface follows a geodesic, spray coating uniformity is better. For a surface of revolution, all longitudes are geodesics, and the thinnest and thickest latitude curves on the surface are also geodesics. If a longitude is used to define the spray trajectory on the surface of revolution, the trajectory generated by the longitude through isometric offset is not a geodesic. If we consider dividing the surface of revolution by using a series of planes at equal angles across the axis of rotation in order to obtain a series of geodesics, the distance between these geodesics along latitudinal direction is not isometric. This indicates that it is difficult to satisfy the requirement for coating uniformity using the conventional spraying method, that is, spray gun move along the meridian at a constant velocity and an equal height. However, if a non-conventional spraying method is used, using variable velocity and variable height: on one hand, it is difficult to set up a spraying deposition thickness model; while on the other hand, it is difficult to realize smooth transitions of the spray gun orientation because of the potential occurrence of self-intersection of the trajectory of the spray gun, this being generated through the offset of initial spray trajectory. However, the distance between the latitudes on the surface of revolution along the longitudinal direction is equal, and the geodesic curvature at every point on the same latitude curve is equal. Therefore, a latitude curve on the surface of revolution can be used as the spray trajectory in order to conform to uniformity of spray coating.

Supposes the parametric equation for a surface of revolution to be $r(u_1,u_2)=(f(u_2)\cos u_1,\ f(u_2)\sin u_1,\ g(u_2))$, $(0\le u_1<2\pi,\ u_a\le u_2\le u_b)$, where $(f(u_2), g(u_2))$ represents a curve on the plane $Oxz$. So, for any point $\boldsymbol{P}$ of the spray trajectory on the surface of revolution, the transformation matrix $\mathbf{T}_P^O$ can be written as:

$$\mathbf{T}_P^O=\begin{bmatrix}\dfrac{f'}{\sqrt{f'^2+g'^2}}\cos u_1 & \sin u_1 & \dfrac{g'}{\sqrt{f'^2+g'^2}}\cos u_1 & f\cos u_1\\ \dfrac{f'}{\sqrt{f'^2+g'^2}}\sin u_1 & -\cos u_1 & \dfrac{g'}{\sqrt{f'^2+g'^2}}\sin u_1 & f\sin u_1\\ \dfrac{g'}{\sqrt{f'^2+g'^2}} & 0 & -\dfrac{f'}{\sqrt{f'^2+g'^2}} & g\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{11}$$

where, $f'$ and $g'$ respectively represent the first-order derivatives of $f(u_2)$, $g(u_2)$ against the parametric variable. Let us take the paraboloid of revolution as a case to illustrate the calculation method. Supposed its equation is $r(u_1,u_2)=(\sqrt{2pu_2}\cos u_1,\sqrt{2pu_2}\sin u_1,u_2)$. Take the parameters $u_{1P}=0$ and $u_{2P}>0$ for the spray center $\boldsymbol{P}(u_1, u_2)$, then any point $\boldsymbol{S}$ $(\tilde{x}_S,\tilde{y}_S,\tilde{z}_S)$ on the surface is $(\sqrt{2pu_{2S}}\cos u_{1S},\sqrt{2pu_{2S}}\sin u_{1S},u_{2S})$. The coordinates of point $\boldsymbol{S}$ under the moving frame of reference of point $\boldsymbol{P}$ can be calculated using Eq.9 -Eq.11.

$$\begin{cases} x_S = \dfrac{\sqrt{2}\left(p\sqrt{u_{2S}}\cos u_{1S}+u_{2S}\sqrt{u_{2P}}-{u_{2P}}^{3/2}-p\sqrt{u_{2P}}\right)}{\sqrt{p+2\tilde{z}_P}} \\ y_S = \dfrac{\sqrt{2p}\left(2{u_{2P}}^{3/2}\sqrt{u_{2S}}\cos u_{1S}-2{u_{2P}}^{3/2}\sqrt{u_{2S}}\sin {u_{1S}}^2\right)}{\sqrt{(p+2u_{2P})u_{2P}}}+\dfrac{\sqrt{2p}\left(-p\sqrt{u_{2S}u_{2P}}\sin u_{1S}-u_{2P}u_{2S}-u_{2P}\right)}{\sqrt{(p+2u_{2P})u_{2P}}} \\ z_S = \dfrac{\sqrt{p}\left(2\sqrt{u_{2P}u_{2S}}\cos u_{1S}-u_{2P}-u_{2S}\right)}{\sqrt{p+2u_{2P}}} \end{cases} \tag{12}$$

$$c_S = \frac{h^2}{(h-z_S)^3}\left(\frac{x_S\sqrt{2p}\left(-\sqrt{u_{2S}}\cos u_{1S}+\sqrt{u_{2P}}\right)}{\sqrt{p+2u_{2P}}\sqrt{p+2u_{2S}}}+y_S\sin u_{1S}-\left(\frac{\sqrt{2u_{2P}}\sqrt{u_{2S}}\cos u_{1S}+p}{\sqrt{p+2u_{2P}}\sqrt{p+2u_{2S}}}(z_S-h)\right)\right) \tag{13}$$

Then Eq.12 and Eq.10 can be used to calculate the coordinates of point $r_S(u_{1S},u_{2S})$ on the paraboloid of revolution for corresponding point $\boldsymbol{F}$ $(x_F, y_F)$ in the elliptical spray area. Eqs.12 is nonlinear and can be solved using numerical methods. The coefficient of spraying orientation in the moving frame of reference is expressed as.

Through the above processes, the following steps are obtained for calculating the continuous spraying deposition thickness $q_S(x)$ along the spray trajectory of the curved surface.

First, the coordinate values $(x_S, y_S, z_S)$ in the moving frame of reference for point $\boldsymbol{S}(u_{1S}, u_{2S})$ on the parametric surface are calculated; Second, the components $(x_F, y_F)$ of point $\boldsymbol{F}$ on the spraying plane $\pi$ corresponding to the point $\boldsymbol{S}$ is determined using Eq.10; Third, coefficient $c_S$ and $\eta_S$ are calculated using Eq.3 and Eq.6 respectively; Then, these parameters are substituted into Eq.4 to calculate $q_S(x)$.

## 4. SPRAY TRAJECTORIES PLANNING FOR A SURFACE OF REVOLUTION

### 4.1 COMPUTATION OF SPRAYING DEPOSITION THICKNESS

Supposed the circle of latitude of the surface of revolution is the spray trajectory, the moving frames of reference are established for two trajectories, as shown in Fig. 4.

$$q(x,w,v) = \begin{cases} q_1(x,v) & x\in ep_1, x\notin ep_2 \\ q_1(x,v)+q_2(x,v,w) & x\in ep_1, x\in ep_2 \\ q_2(x,v,w) & x\in ep_2, x\notin ep_1 \end{cases} \tag{14}$$

$$q_\alpha = \frac{1}{v_\alpha}\int_0^{t'} q_u\left({x'_\alpha}^2\right)^{\beta_1-1}\left(1-\frac{(bx'_\alpha-t'\eta_\alpha)^2}{b^2{x'_\alpha}^2}\right)^{\beta_2-1} c_\alpha dt' = \frac{1}{v_\alpha}c_\alpha q_u\left({x'_\alpha}^2\right)^{\beta_1-1}\sum_{i=1}^{n+1}\left(1-\frac{(bx'_\alpha-\eta_\alpha t'_i)^2}{b^2{x'_\alpha}^2}\right)^{\beta_2-1}\frac{T_\alpha}{n} \tag{15}$$

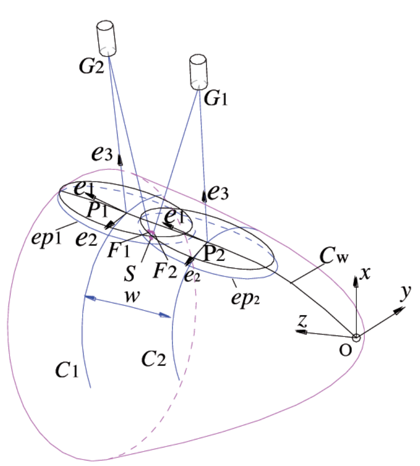



*Fig.4: Spray trajectories and moving frame for a revolved surface*

The spraying deposition thickness between the two trajectories is.

where $x$ is the coordinate along vector $\boldsymbol{e}_1$ direction in the moving frame. The cumulative spraying deposition thickness along the trajectories can be obtained by Eq.4-Eq.5.

where, $\alpha=1,2$ $x'_1=\sqrt{1-\frac{x_F^2}{a^2}}$, $x'_2=\sqrt{1-\frac{(x_F-w)^2}{a^2}}$, and $w$ is the distance between the two trajectories, $t'_i=\frac{T'}{n}(i-1)$, $i=1, 2, 3, \ldots, n+1$, $T'=2b\left(1-x^2/a^2\right)^{1/2}$, $c_1$ and $c_2$ are respectively coefficients of spraying orientation of trajectory-1 and trajectory-2 at point $\boldsymbol{S}$.

### 4.2 OPTIMIZATION OF THE SPRAY TRAJECTORIES

The objective of spray trajectories planning is to minimize the deviation between the practical spray thickness and the ideal coating thickness which satisfies the relevant technical requirements. The spray gun velocity $v$ and the trajectory space $w$ are the optimized parameters. According to literature [14] shown that, with the air spray gun at a velocity of $v$=0.128m/s, a height of $h$=220mm from the plane $\pi$, the resulting parameters are $a$=121.99mm, $b$=46.57mm, $q_u$=22.32µm, $\beta_1$=2.470 and $\beta_2$=4.720. Assumed the ideal spray thickness is $q_d$ = 0.8mm, the least square method can be used to work out the spray gun velocity $v$ , and trajectory space $w$. The objective function is:

$$\min E(x,w,v)=\int_0^w (q(x,w,v)-q_d)^2 dx \tag{16}$$

For Eq.15 suppose

$$\rho(x,w)=c_\alpha q_u\left(x_\alpha'^2\right)^{\beta_1-1}\int_0^{t'}\left(1-\frac{(bx_\alpha'-\eta_\alpha t')^2}{b^2 x_\alpha'^2}\right)^{\beta_2-1}dt'$$

Then we get

$$q=\frac{1}{v}\rho(x,w) \tag{17}$$

Substituting Eq.17 into Eq.16, we have

$$\frac{\partial E(v,w,r)}{\partial v}=0 \tag{18}$$

By solving Eq.18 for spraying velocity $v$, we get

$$v(w)=\frac{\int_0^w \rho^2(x,w)dx}{q_d\int_0^w \rho(x,)dx} \tag{19}$$

Substituting Eq.19 into Eq.17 we can obtain that the objective function of the optimized trajectory for the surface of revolution is related to just the trajectory space $w$. Then the objective function in Eq.16 is a single objective optimization problem. For the one-dimensional optimization problem, here we use the golden section method and parabolic interpolation method to calculate the optimized parameter.

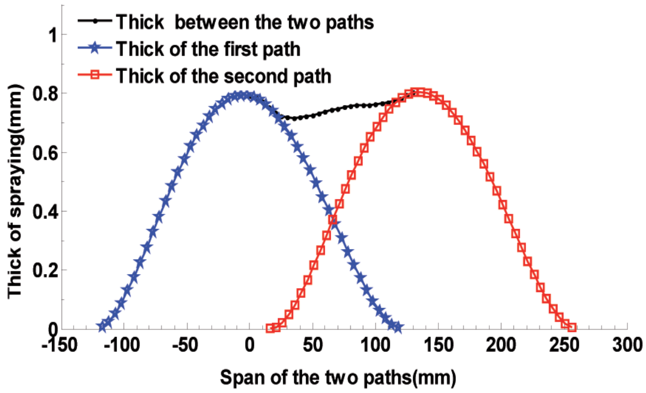



*Fig.5: Spraying deposition thickness of the two trajectories and the thickness between them*

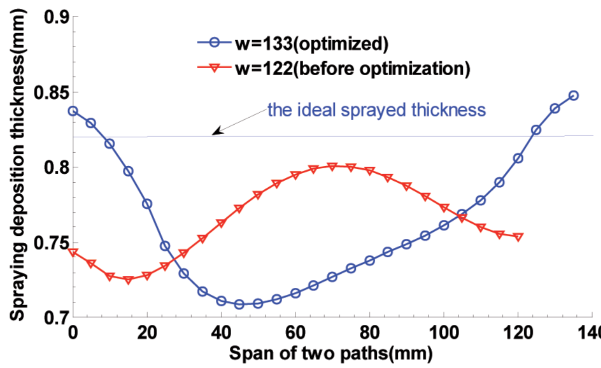



*Fig. 6: Spraying deposition thickness between two trajectories before and after optimization*

### 4.3 VERIFICATION SIMULATION

In order to illustrate the whole calculation process, the space w between two spray trajectories can be calculated for the Matlab programming environment. Assumed the parameter of the paraboloid of revolution as $p$=80 to calculate, the first spray trajectory is on the latitude circle of $u_{2s1}$=400mm, and supposes geodesic distance $w$ along the longitudinal direction as the optimized variable, after optimization computation, the space between the two trajectories is $w$=133.22mm. Substituting $w$ into the arc length equation, the second trajectory is obtained to be the latitude circles of $u_{2s2}$=527.8mm. And the sum of the square of deviations is $E$=0.0478 with a spray gun velocity of $v$=0.0983m/s. The simulation result is shown in Fig.5.

Supposed that the space of the two trajectories before optimization is 122mm, which is equal to the half of the long axis of the spray area, the calculation deviation of spraying deposition thickness between the two spray trajectories is 0.1807 with the spray gun velocity 0.157m/s. The numerical simulations show that the optimization deviation is lower than that before optimization, as shown in Fig.6.

## 5. DISCUSSION AND CONCLUSION

(a) Deposition model of spray-coating at a point of the curved surface shows that the spraying deposition thickness is not only related to deposition model of the plane, but also related to the coefficient of spraying orientation of the point, which indicates mapping of deposition thickness from spraying plane to curved surface and makes the computation of spraying deposition thickness at any point on the surface simple and effective.

(b) When the spray gun travels at a high and constant velocity, the most feasible spray trajectories of a parametric surface of rotation are latitude circles. The continuous spraying deposition thickness on the curved surface is not only related to the deposition thickness in a unit time, but also related to the spray gun velocity and speed ratio coefficient. This indicates the rate of accumulation of coating thickness between the center and the other point within the sprayed region.

(c) For a paraboloid of revolution, the least squares method can be used to optimize the spray trajectory space and the spray gun velocity. It performs as well as the specific equations for calculating. The golden section method and parabolic interpolation method are used to calculate the optimization parameter, the simulation results show that the optimization calculation on the surface of revolution is more effective compared with the results without optimization. In the future, we will consider spray trajectory planning on compound surface, which is a multi-objective and complicated problem. Also genetic algorithms and neural networks in general the inverse methods will be discussed and considered.

(d) The proposed computing method and steps in the determination of spray trajectories based on the moving frame can be applied to regular parameterized surfaces, such as extruded faces and revolved surfaces etc. It will promote painting quality. Certainly, it still needs further efforts on application research.

## APPRECIATION

This work was supported by the Project funding of The Natural Science Foundation of Gansu Province 1308RJZA134.

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