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FRACTAL GEOMETRIES DESIGN FOR THREE-DIMENSIONAL PRINTING: ANALYSIS OF PERFORMANCE AND VIBRATION RESPONSE FOR PRODUCT DESIGN

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ABSTRACT:

Additive Manufacturing (AM) has enhanced the development of complex designs and geometries that are difficult or impossible to achieve with conventional manufacturing methods. Therefore, research focused on the analysis of the functional properties related to the behavior conferred by the filling material and the internal structure became especially relevant. In this sense, fractal geometry is one of the most established theories to characterize complex geometries based on repetitive self-similar patterns that allow complex shapes to be described and relate them to mechanical, rheological and morphological properties. The design of fractal microstructures promises the development of advanced materials with improved mechanical properties and multiple functions related to vibration absorption, weight reduction, among others. In relation to the compatibility of elastic materials with three-dimensional printing processes, the study of fractal geometries in terms of geometric design and its correlation with improved mechanical behavior is established as an objective. Several TPU-based fractal samples were designed and tested using compression processes to characterize the behavior of elastic fractal geometries as a function of fractal dimension. Specifically, Sierpinski pyramids-based specimens were designed to explore the influence of fractal dimensions and iterations on the elasticity of the samples. Additionally, a case study was carried out to by using Finite Element Method (FEA) in order to analyze the stress distribution and vibrational behavior based on fractal order. The results demonstrated a notable variation in compression behavior and vibrational behavior depending on the fractal order, highlighting the potential of fractal geometries to enhance mechanical performance in AM applications.

Keywords: Fractal Geometries, Product Design, Three-Dimensional Printing, Performance, Thermoplastic Polyurethane.

1.- INTRODUCTION

Additive Manufacturing (AM) has enhanced the development of complex designs and geometries that are difficult or impossible to achieve with conventional manufacturing methods. The wide range of materials compatible with AM processes has increased the use of elastomeric polymer materials in recent years. Especially thermoplastic polyurethanes (TPU) have proven to be a popular choice among different elastic materials [1–5]. Literature reports that TPU seems to be a good choice in terms of softness, biocompatibility, and potential energy generation for original shape recovery in rigid and elastic objects [6–8]. Furthermore, the performance of TPU was shown to be proportional to the filler density independently of the sample thickness, establishing the suitability of this material for deformation and energy absorption applications [9, 10]. In this sense, the literature documented that TPU would allow specific properties to be obtained in terms of texture, flexibility and other sensations [10–14], This is achieved from the geometric and manufacturing properties of the printed object, which would allow the elasticity to be adjusted in a customized way to all TPU-based products applications [14–16]. Consequently, research focused on the analysis of the functional properties related to the behavior conferred by the filling material and the internal structure of complex structures became especially relevant [6, 17]. In this sense, conventional manufacturing methods currently do not rival AM processes such as Fused Filament Fabrication (FFF) because it is possible to generate free-support non-solid internal geometries with the possibility of providing a different elasticity to the same product, as well as obtaining elastic materials from rigid materials and different elasticities from the same elastic material [5–7]. In concordance, stress concentrators play a critical role in defining complex structures. Stress concentrators are critical regions in 3D printed structures where stress levels are significantly higher than the average stress [18]. These areas are often the starting points for failure, making it essential to understand them for designing reliable 3D printed components [19]. Accordingly, AM process parameters such as layer thickness and orientation can influence the formation and behavior of these concentrators [20]. Complex structure design theory indicates that lattice structures can be optimized to minimize stress concentrations by distributing stress evenly throughout the material

[21]. Furthermore, research on 3D-printed fractal structures documented that these geometries effectively manage stress distribution, reducing the risk of failure under impact or loading conditions [20, 22].

According to 3D-printed fractal structures, fractals is one of the most established theories to characterize complex geometries based on repetitive self-similar patterns that allow complex shapes to be described and relate them to mechanical, rheological and morphological properties [23, 24]. Scientific evidence has documented an interesting correlation between the effective density of structures based on fractal geometries and their mechanical properties [25–27]. Therefore, the design of fractal microstructures turns out to be promising from the perspective of the development of advanced materials with improved mechanical properties and multiple functions [28]. Multiple levels of hierarchy have been analyzed in terms of different fractal shapes and orders, suggesting that different performances can be obtained depending on the effective density of the resulting structures [29].

For this reason, this work aims to characterize the geometry of additively manufactured elastic samples based on the fractal dimension. In this sense, several samples based on Sierpinski pyramids have been designed, in order to explore the influence of fractal dimensions and iteration on the elasticity of the designed elastic samples. Likewise, based on the results obtained, it is intended to carry out a case study based on sample's vibrational behavior through finite element analysis (FEA). The objective is to analyze the stress behavior and vibrational modes in function of their fractal dimensions to establish hypotheses about their application in product design.

2.- MATERIALS AND METHODS

1.1. Samples Design

The design of the samples was based on Sierpinski pyramids to explore the influence of fractal dimensions and iterations on the elasticity of the samples. The sample designs varied according to the fractal order (See Fig. 1).

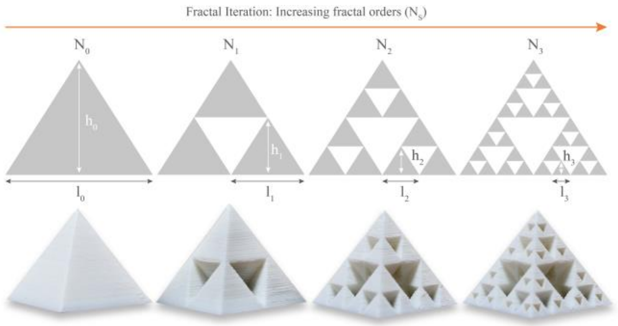

Fig. 1. Fractal iterations of Sierpinski Pyramid-based samples

The dimensions established for each Sierpinski pyramids-based sample according to the fractal order can be seen represented in Table 1. Cross sectional dimensions were originally obtained from a prism with a 50 mm square base, where the height was set to 35 mm in order to meet the fundamentals of supportless printing.

Fractal order	H_s(mm)	L_s(mm)	Weight (gr)
N_0	35	50	31
N_1	17.5	25	25
N_2	8.75	12.5	19
N_3	4.375	6.25	15

Table 1. Samples dimensions.

1.2. Material

The manufacturing material of the fractal samples was the commercial Filaflex® 95A TPU from Recreus. This material has a diameter of 1.75 mm and the main mechanical properties are detailed in Table 2.

Material	Filament Diameter (mm)	Density (g/cm3)	Tensile Strength (MPa)	Printing Temperature (°C)	Shore A Hardness
TPU95A	1.75	1.08	55	215 - 250	95A

Table 2. Material properties.

1.3. Samples' production

The fractal samples were manufactured on the FFF Creality 3D Ender-3® machine. The machine used contained modifications to improve the printing of elastic filaments. The main manufacturing parameters can be seen represented in Table 3.

Parameters	*Value*
Layer height	*0.2 mm*
Layer width	*0.4 mm*
Printing temperature	*220 ºC*
Bed temperature	*35 ºC*
Printing speed	*20 mm/s*
Displacement speed	*180 mm/s*
Retraction	*enabled*
Retraction distance	*1.5 mm*

Table 3. Manufacturing Parameters

1.4. Procedure of evaluation

Compression tests were carried out on the manufactured samples to characterize samples' elasticity based on the fractal order. According to [30], compression speed was set at 1 mm/min. The tests were conducted using the Shimadzu Autograph testing machine, model AG-X series 101. Also, the Trapezium® universal software, was used to manage all test variables.

The loading direction on the samples was kept parallel to the printing direction to control possible variations in elasticity arising from the deformation mode governing in that direction [31]. The lower plate was fixedly positioned while a displacement of the upper plate was established until approximately 70% compression of the samples (see Fig. 2).

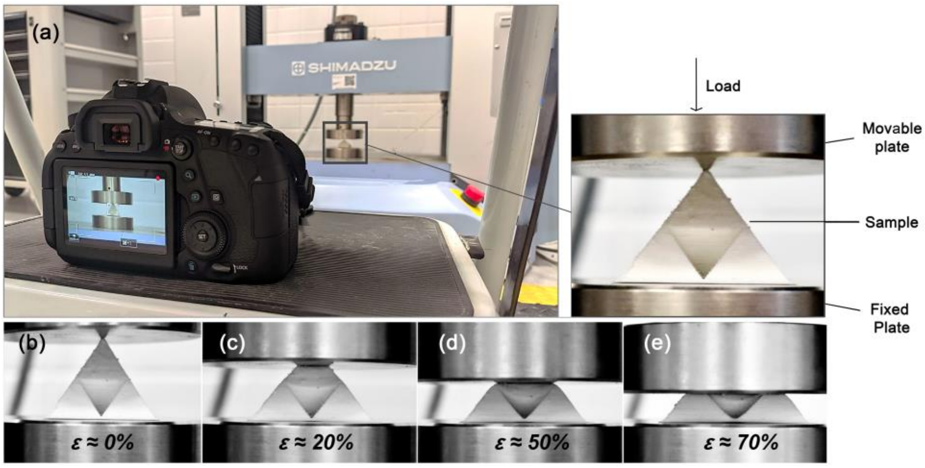

Fig. 2. Compression testing. (a) Experimental Set up; (b) 0% sample compression; (c) 20% sample compression; (d) 50% sample compression; (e) 70% sample compression.

The results derived from the compression tests through engineering stress-strain curves were analyzed and graphically plotted to quantify the Elastic modulus (E) of the samples. In this sense, E indicates the relationship between the stress and strain values during compression of the samples and was obtained through its equivalence with the slope of the initial elastic region of the stress-strain curve using Eq. 1 [32–34]:

$$E = \frac{d\sigma}{d\varepsilon}\Big|\, \varepsilon < \varepsilon_y \quad (1)$$

Where stress "σ" was obtained from the compression test data through Eq. (2):

$\sigma = \frac{F}{S}$ (2)

Where the axial load applied to the sample corresponded to "F" and the cross-sectional area on which the axial load was exerted corresponded to "S". To determine the cross-sectional area, an approximation was made based on the deformation "ε", which was obtained using Eq. 3:

$\varepsilon = \frac{\Delta L}{L_o}$ (3)

Where, the initial length of the sample "Lo" is equivalent to the height of the fractal specimens and equal to 35 mm. Also, the measurement difference between the initial distance and the distance at the measured point corresponded to "∆L". In this sense, for a deformation range of 5%-100%, "S" would range approximately between 2.5 mm - 50 mm, where the 50 mm are related to the measurements of the base of the samples based on Sierpinski pyramids.

1.5. FEA Analysis

As mentioned above, a case study is carried out focused on analyzing the vibrational behavior of fractal samples. Specifically, the FEA analysis covered the analysis of the vibration modes of the fractal samples according to [35].

Vibrational analysis response was evaluated using FEA analysis in SolidWorks®. Since FEA analysis cannot account for manufacturing parameters, to accurately approximate the effects of manufacturing parameters on FEA results, a simulation material sample replicating the primary mechanical properties of TPU 95A was designed based on previously characterized samples. The objective was to reduce possible variations due to the anisotropic characteristics resulting from the FFF process.

In this case study, Sierpinski pyramid-based samples were used as product foot pads. A top plate was modeled to simulate the product resting on the fractal samples. The simulation setup involved positioning a fractal sample, in an inverted orientation, at each corner of the plate. For the vibrational analysis, the boundary conditions were defined by fixing the flat faces of the fractal samples in contact with the upper plate, while a gravity force was applied to the top of the plate (See Fig. 3). The objective is to compare the vibration isolation capacity of fractal samples. The range of vibrations studied is set around 3 Hz. This range of average vibrations is slightly higher than the vibrations produced by means of transport, and is within the range of hertz related to the use of machinery dedicated to industrial activities [36]. Prior to the vibrational analysis, a study of the stress behavior of the fractal samples was conducted to understand stress accumulation as a function of the fractal order. In this sense, a gravity force was applied to the upper flat surface of the samples, with the apex of the Sierpinski pyramid-based samples set as fixed (See Fig. 4).

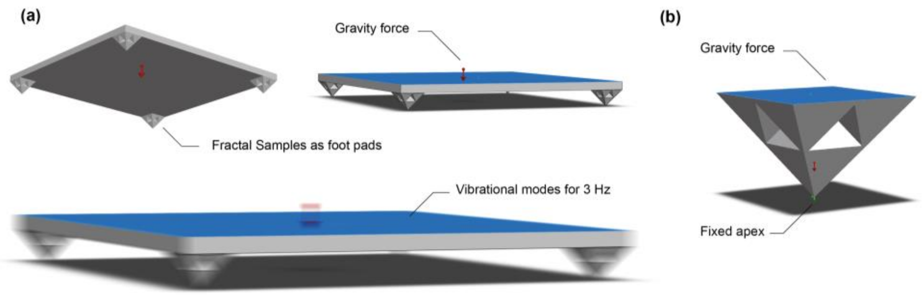

Fig. 3. FEA setup. (a) Vibrational Study; (b) Stress behavior

3.- RESULTS AND DISCUSSION

3.1. Samples' characterization

The elastic region at the beginning of plastic collapse, the progressive failure and the densification modes corresponding to the usual deformation values in strain-stress curves can be seen represented (See Fig.4). The Elastic modulus was calculated from the initial slope of the stress-strain curve. The fractal samples exhibited elastic behavior up to approximately 10% strain. However, due to possible initial fluctuations of the elastic material [6], the stiffness values for each of the samples were obtained from the range of recorded stresses between 5% and 10% strain in the elastic region of the stress-strain curves. On the other hand, the plateau region progressively increased from approximately 10% compressive displacement until reaching a strain of approximately 55%-60% before densification. According to [37], the onset of the plateau region was determined in the yield strength region at the end of the initial slope of the stress-strain curve, where the material experienced considerable plastic deformation without a significant increase in the applied stress. Similarly, the beginning of the densification period was established after the plateau region, just at the start of the significant increase in the slope of the stress-strain curve.

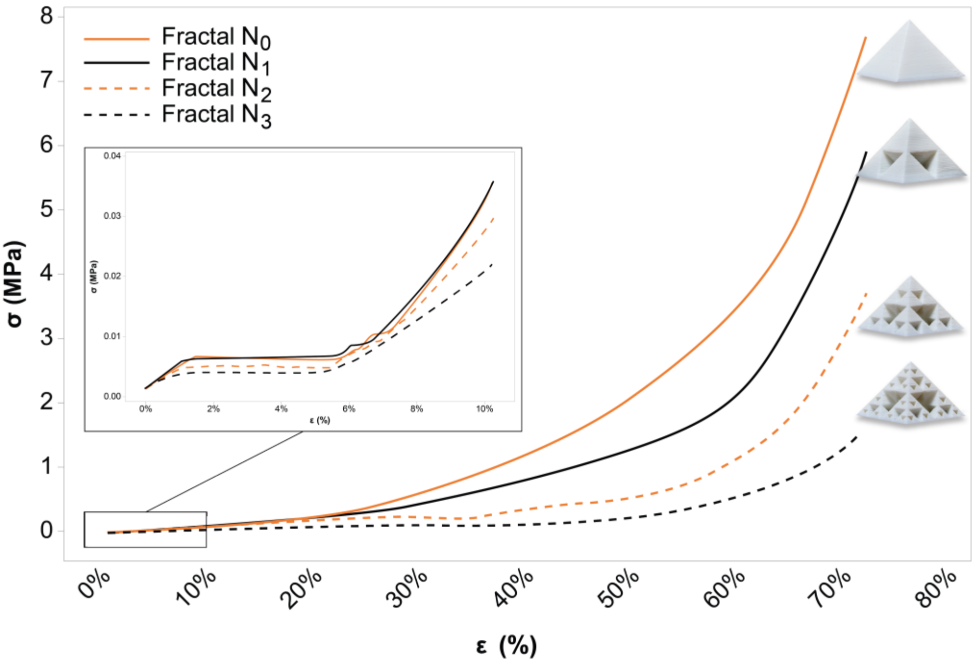

Fig. 4. Strain-stress curves.

Several effects were observed on the elasticity of the specimens due to the fractal order. Specifically, higher strain values were shown by samples with higher fractal order compared to samples with lower fractal order. In this sense, the densification period of higher fractal order samples tends to occur at approximately 60% deformation, while for the rest of the samples this percentage tends to be reduced by approximately 5%-10%. Similarly, samples of lower fractal order showed higher densification stress values compared to those of higher fractal order. In this sense, it has been documented that the size and number of pyramids influence densification and plateau stresses. Consequently, the slope in these regions turns out to be greater as the density of the specimens increases, which is equivalent to the size and number of the pyramids. This would explain why a greater slope is always observed for samples of lower fractal order.

In the same way, Elastic modulus values have been compiled for all the specimens represented along with the fractal order (See Fig.5). In general, a linear decrease in stiffness is observed with respect to the increase in fractal order. This could be due to the variation in the samples' density, where the specimens of higher fractal order are much more porous and have less volume of material with respect to the total volume, increasing the stiffness [15]. However, it is observed that the stiffness of the N1 sample is very similar to the stiffness of the N0 sample. In this sense, the linear decrease of the density of the specimens with the increase of the fractal order it is not equated to the decrease in stiffness (See Fig.5). Therefore, it is possible to obtain similar stiffness in the N0 and N1 samples with a consequent saving of 20% in material consumption and lower manufacturing times. This result can be seen documented in other recent research, where complex structures offered similar stiffness results to solid structures [21]. This phenomenon may derive from the localized accumulation of tensions produced by the geometry of the structure itself, which significantly affects the stiffness .

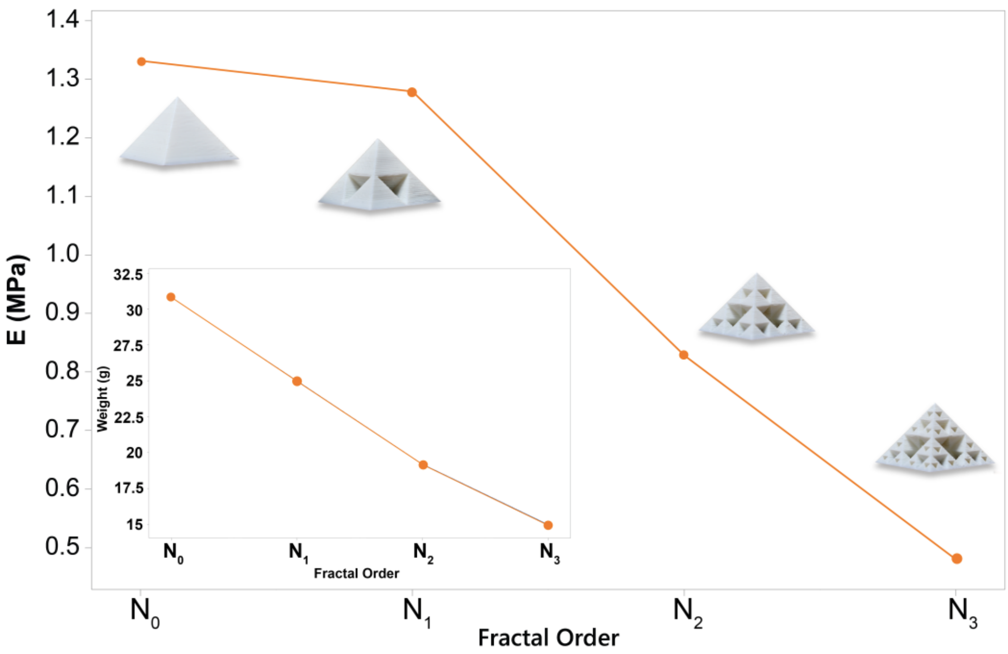

Fig. 5. Elastic modulus variation.

3.2. FEA analysis

According to the similar behavior obtained between samples N0 and N1 with different fractal order, the FEA analysis has been focused exclusively on both samples.

It is observed that the stress distribution patterns vary significantly between N0 and N1 samples (See Fig. 6). For fractal sample N0, stress accumulation is observed at the apex of the pyramidal geometry, which is expected since it is the main point of contact of the fractal sample with the floor surface. However, for fractal sample N1, stress accumulation is primarily found along the inner edges of the fractal geometry, particularly the diagonal edges. This suggests that the inner edges of geometry N1 act as stress concentrators [18]. This result is consistent with recent research where experimental and simulation models documented an increasing stress accumulation at the intersection between vertical and horizontal surfaces in complex structures [38, 39].

Quantitative stress analysis reveals that sample N1 exhibits both the lowest and highest stress levels compared to sample N0. This suggests that stress scaling is more pronounced in sample N1, likely due to the influence of stress concentrators, which can significantly impact stress distribution [22, 38]. Consequently, despite being a lighter structure, sample N1 demonstrates similar stiffness to sample N0.

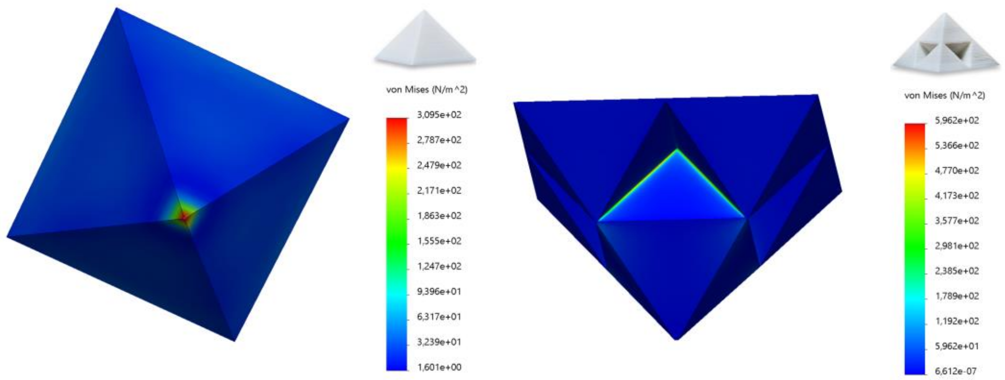

Fig. 6. Stress behavior.

Accordingly, the vibrational modes presented by the samples turn out to be similar. See the direct comparison between the same vibrational mode for both samples for an approximate frequency of 3 Hz (See Fig. 7). It is observed that the vibration area in the center is smaller tending to two facing corners. Likewise, analyzing the range of amplitudes it is observed that the upper limits (red color) are similar in both samples. However, with respect to the lower limits (blue color) it is observed that N1 has lower amplitudes. Therefore, it is possible to conclude that despite having a lower fractal order, sample N1 manages to reduce the vibrations in the facing corners. Therefore, there are indications of similar vibrational behavior despite the sample being more efficient in terms of material use and manufacturing times.

This could be attributed to the stiffness of the samples. Recent research has documented that the vibrational frequencies of reticular structures are proportional to the volume fraction and the size of the repetitive structural unit. In this context, stiffness plays a critical role since complex repetitive structures can exhibit higher natural frequencies due to increased stiffness [40]. This would explain the results observed in the FEA analyses.

From the perspective of fractal theory, the use of fractals could be focused on understanding the impact of fractal order and the size of the repetitive unit on stiffness and relative density. This approach could lead to improved or customized vibrational behavior by optimizing these parameters.

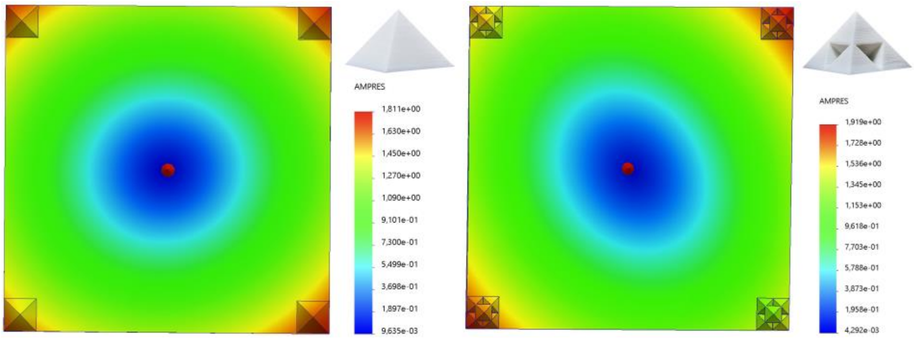

Fig. 7. Vibrational analysis.

3.3. Limitations and further research

This work has several limitations. Firstly, only the vibrational behavior was investigated by FEA analysis. Likewise, with regard to the 3D printing of the fractal samples, the study of the vibrational behavior of the fractal samples will be carried out empirically and compared with the FEA results [35]. In this way, comparative efficiency analysis of the studied samples could be conducted.

Secondly, the study analyzed the vibrational behaviour of only two fractal orders based on empirical compression results. All fractal orders should be fully explored in a future study for the analysis and detection of patterns in vibrational behaviour.

Third, the deformation process of the samples could be analyzed in order to validate existing results or to derive new ones.

Finally, the study analyzed a thermoplastic polyurethane material, where elasticity play an important role in stiffness [49]. In this sense, the effects of non-elastic materials may have could be explored, as well as different Shore A hardness TPU's.

4.- CONCLUSIONS

The results obtained documented that for low and medium strain values, fractal samples exhibit similar behavior under quasi-static compression, despite of fractal order. Significant differences between the samples were observed during plateau and densification region. This suggests that higher fractal order samples may perform comparably to lower fractal order samples in strain-limited applications, likely due to the stress distribution within the fractal geometry.

FEA stress analysis revealed that stress tends to concentrate along the inner edges of the Sierpinski pyramid-based specimens. Particularly, the diagonal edges exhibited greater stress concentration, suggesting that these internal edges function as stress concentrators. Quantitative stress analysis showed that sample N1 exhibited both lower and higher stress levels compared to sample N0, suggesting more pronounced stress scaling in sample N1. This accounts for the similar stiffness observed in both samples despite the difference in fractal order.

Moreover, the analysis of vibrational behavior corroborated that higher-order fractal structures can offer similar vibrational performance to lower-order structures, while being more efficient in material use and manufacturing times. These findings can be attributed to the effective density and stiffness of the samples, explaining the FEA results.

From a fractal theory perspective, optimizing fractal order and repetitive structural unit can improve or customize vibrational behavior, providing unique advantages in geometric flexibility and design freedom. For example, applications such as lightweight supports or footpads for transportable incubators manufactured by 3D printing could benefit from the enhanced vibrational performance and material efficiency of fractal geometries. This approach leverages the advantages of vibrational behavior while reducing material usage and energy costs through optimized manufacturing processes enabled by fractal geometries.

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